M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

PREPARED BY
DR. AMARESH RANJAN
Assistant Professor
Nalanda Open University
Patna
Contact No. : 9279177009

SET, RELATIONS AND FUNCTIONS

Objective

Introduction of Factorial Notation

Circular Permutation

Combination

Objective

Counting is based on elementary combination, which is a branch of discrete mathematics. Including concepts, such as factorial notation, permutations and combinations. It is Important to study the basics of counting and combinatorics preliminaries for understanding the concep of combinatorics.

Basics of counting can be divided into two categories viz Rule of Addition and Rule of Multiplication.

In permutation we study the arrangement of order but in combination we study the selection or grouping or choosing of things.

Introduction :

In this unit you will be learn about the theory of counting. If two tasks T_1 and T_2 can be Performed in n_1 and n_2 ways, respectively and these are not to be performed simultaneously, then two task T_1 and T_2 can be performed in $n_1 + n_2$ ways.

This is the rule of addition.

This unit will also deal with the rule of multiplication. If a task T_1 can be performed in n_1 ways and independent of this task, the second task T_2 can be performed in n_2 ways, so that these two tasks when combined can be performed in $n_1 \times n_2$ ways.

Factorial Notation :

The product of n consecutive positive integers beginning with 1 is denoted by $\underline{|n}$ or n! and read as facorial n.

i.e.

$$\boxed{\underline{|n} \quad n! \quad n(n-1)(n-2)(-3).......4 \quad 3 \quad 2 \quad 1}$$

$$\underline{|4} \quad 4 \quad 3 \quad 2 \quad 1 \quad 24$$

$$\underline{|5} \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 120$$

$\lfloor\underline{6}\quad 6\quad 5\quad 4\quad 3\quad 2\quad 1\quad 720$

$\lfloor\underline{7}\quad 7\quad \lfloor\underline{6}\quad 7\quad 6\quad \lfloor\underline{5}$

$\lfloor\underline{8}\quad 8\quad \lfloor\underline{7}\quad 8\quad \lfloor\underline{8-1}$

$$\boxed{\lfloor\underline{9}\quad 9\quad \lfloor\underline{9-1}}$$

Similarly $\boxed{\lfloor\underline{n}\quad n\lfloor\underline{n-1}}$

Note : (a) $\lfloor\underline{n}$ is defined for positive integer only

(b) $\boxed{\lfloor\underline{0}\quad \lfloor\underline{1}\quad 1}$

Rules of Addition and Rule of Multiplication :

In combinatorial analysis we intend to determine the number of logical possibilities of occurrence of events without looking into individual cases.

(a) Rule of Addition :

Suppose the tasks cannot be performed simultaneously and also suppose that T_1 can be performed in n_1 ways and T_2 can be performed in n_2 ways. Then two tasks T_1 anc T_2 can be performed in $n_1 + n_2$ ways.

In general, suppose a task T_1 can be performed in n_1, ways and second task T_2 in n_2 ways, a third task in n_3 ways and soon and if no two tasks can be performed simultaneously.

Then one of the tasks can be performed in $n_1 + n_2 + n_3$ ways.

(b) Rule of Multiplication :

Suppose a task T_1 can be performed in n_1 ways and independent of this task, the second task T_2 can be performed in n_2 ways so that these two tasks when combined can be performed in $m \times n$ ways. In general suppose a task T_1 can be performed in n_1 ways and following T_1, a second task T_2 can be performed in n_2 ways and following task T_2 a third task T_3 can be performed in n_3 ways and so on then all k tasks can be performed in the sequence $T_1, T_2, T_3, \ldots\ldots\ldots T_k$ in exactly $n_1, n_2 \ldots\ldots\ldots\ldots\ldots\ldots n_k$ different ways.

Permutation

Each of the different arrangement which can be made by taking some or all of a number of given things at a time is called permutation.

The number of ways of arrangements of n different things taken r at a time. and it is written as ${}^nP_r$

i.e. $\boxed{{}^nP_r \ \frac{|\underline{n}}{|\underline{n-r}}, n \_ r}$

Where n = Total number of given things

r = taken at a time.

P = permutation.

The following are some of the cases of permutation.

(a) Formation of numbers with given digits

(b) Formation of word with given letters.

(c) Arrangement of persons in a row or at a round table.

(d) Arrangement of books on a shelf.

Find the value of

(a) $${}^7P_3 \quad \frac{|\underline{7}}{|\underline{7 \quad 3}} \quad \frac{|\underline{7}}{|\underline{4}} \quad \frac{7 \quad 6 \quad 5 \quad |\underline{4}}{|\underline{4}} \quad 210$$

(b) $${}^8P_8 \quad \frac{|\underline{8}}{|\underline{8-8}} \quad \frac{|\underline{8}}{|\underline{0}} \quad \frac{|\underline{8}}{1} \quad |\underline{8}$$

$$|\underline{8} \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 40320$$

(c) $\quad 9P_{10} \quad \frac{|\underline{9}}{|\underline{9 \quad 10}} \quad \frac{|\underline{9}}{|\underline{\ \ 1}}$ not possible because r > n.

If $^{n}P_5 \quad 20 \quad ^{n}P_3$ find the volume of n.

Solutions :

$$^{n}P_5 \quad 20 \quad ^{n}P_3$$

$$\frac{|\underline{n}}{|\underline{n-5}} \quad \frac{20 \quad |\underline{n}}{|\underline{n-3}}$$

$$\frac{1}{|\underline{n-5}} \quad \frac{20}{(n-3)(n-4)|\underline{(n-5)}}$$

(n–3) (n–4) = 20

n^2 – 7n + 12 = 20

n^2– 7n + 12–20 = 0

n^2 – 7n –8 = 0

n^2 – 8n +n – 8 = 0

n(n – 8) + 1 (n – 8) = 0

(n + 1) (n – 8) = 0

$$n \quad 1 \quad 0 \quad n-8 \quad 0$$

$$(n \quad -1) \quad (n \quad 8)$$

Thus n is a positive integer.

Uf $^{10}P_{n-1} : {}^{11}P_{n-2} \quad 30:11$ find the value of n.

Solution :

$$\frac{^{10}P_{n-1}}{^{11}P_{n-2}} \quad \frac{30}{11}$$

$$\frac{\dfrac{\underline{|10}}{\underline{|10-(n-1)}}}{\dfrac{\underline{|11}}{\underline{|11-(n-2)}}} \quad \frac{30}{11}$$

$$\frac{\underline{|10}}{\underline{|10-(n \quad 1)}} \quad \frac{\underline{|11-n \quad 2}}{\underline{|11}} \quad \frac{30}{11}$$

$$\frac{\underline{|10}}{\underline{|11-n}} \quad \frac{(13-n)(12-n)\underline{|11-n}}{11 \quad \underline{|10}} \quad \frac{30}{11}$$

(13 –n) (12 – n) = 30

$156 - 25n + n^2 = 30$

$n^2 - 25n + 156 - 30 = 0$

$n^2 - 25n + 126 = 0$

$n^2 - 18n - 7n + 126 = 0$

n(n – 18) – 7(n–18)= 0

(n – 18) (n – 7) = 0

n – 7 = 0

n = 7

Putting the value of n

$10P_{n-1} \quad 10P_{7-1} \quad 10P_6$

$n \quad 7$

But n = 18

$10P_{18-1} \quad 10P_{17}$ not possible because r > n.

Question : How many numbers can be formed by taking 3 digits at a time out of 1, 3, 5, 7, 9 (no digits being repeated in the same number) ?

Solutions : Number of distinct digits = 5

number of digits to be taken = 3

The number of arrangement

$$^{5}P_{3} \quad \frac{\underline{|5}}{\underline{|5-3}} \quad \frac{\underline{|5}}{\underline{|2}}$$

$$\frac{5 \quad 4 \quad 3 \quad \underline{|2}}{\underline{|2}} \quad 60$$

Ques : How many different word of 3 letters can be formed out of the letters of the word PRODUCT ?

Solution :

There are 7 distinct letters in the word PRODUCT. All letters are different we hae to taken 3 at a time.

Total number of arrangement = $^{7}P_{3}$.

$$\frac{\underline{|7}}{\underline{|7-3}} \quad \frac{\underline{|7}}{\underline{|4}}$$

$$\frac{7 \quad 6 \quad 5 \quad \underline{|4}}{\underline{|4}} \quad 210$$

Ques : How many words can be formed from the letters of word ‘PRODUCT’ provided at least three letters appear ?

Solutions :

There are 7 different letters in the word ‘PRODUCT’

There is canditions atleast three letters appear

Case I : Taken 3 letters at a time i.e $^{7}P_{3}$

Case II : Taken 4 letters at a time ${}^7P_4$

Case III : Taken 5 letters at a time ${}^7P_5$

Case IV : Taken 6 letters at a time ${}^7P_6$

Case V : Taken 7 letters at a time ${}^7P_7$

Total number of words taken at least 3 letters at a time.

$= {}^7P_3 + {}^7P_5 + {}^7P_6 + {}^7P_7.$

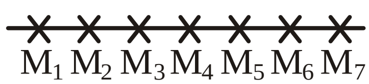
$= 210 + 840 + 2520 + 5040 + 5040$

$= 13650$ Ans.

In how many ways 7 men and 7 women can stand in a straight line so that no two women stand together.

Solution :

If 7 men and 7 women stand in a row then according to Question there are 8 empty places marked with X are possible.



$M_1, M_2, \ldots\ldots\ldots\ldots M_7$ represent the places for 7 men.

7 men can stand in $7P_7$ ways and 7 women can stand in $8P_7$ ways.

The required number of ways

$$(7P_7 \quad 8P_7) \text{ ways}$$

$$\frac{\underline{|7} \quad \underline{|8}}{\underline{|8-7}} \qquad \frac{\underline{|7} \quad \underline{|8}}{\underline{|1}}$$

$$\underline{|7} \quad \underline{|8} \text{ ways.}$$

Find the value of r if ${}^7P_r = 42$.

Solution :

We have

$$^{7}P_r \quad 42 \quad 7 \quad 6$$

$$^{7}P_r \quad \frac{7 \quad 6 \quad \underline{|5}}{\underline{|5}}$$

$$^{7}P_r \quad \frac{\underline{|7}}{\underline{|5}} \quad \frac{\underline{|7}}{\underline{|7-2}}$$

$$^{7}P_r \quad {}^{7}P_2$$

r = 2

$^{m+n}P_2 \quad 90$ and $^{m-n}P_2 \quad 30$ find the value of m and n.

Solution :

We have

$$^{m\ \ n}P_2 \quad 90 \quad 10 \quad 9 \quad \frac{10 \quad 9 \quad \underline{|8}}{\underline{|8}}$$

$$^{m\ \ n}P_2 \quad 90 \quad \frac{\underline{|10}}{\underline{|8}} \quad \frac{\underline{|10}}{\underline{|10-2}}$$

$$^{m\ \ n}P_2 \quad {}^{10}P_2$$

m + n = 10 ---------------- (i)

$$^{m-n}P_2 \quad 30 \quad 6 \quad 5 \quad \frac{6 \quad 5 \quad \underline{|4}}{\underline{|4}}$$

$$^{m-n}P_2 \quad \frac{\lfloor 6}{\lfloor 6-2} \quad {}^{6}P_2$$

$$^{m-n}P_2 \quad {}^{6}P_2$$

m – n = 6 --------------- (ii)

from (i) and (ii)

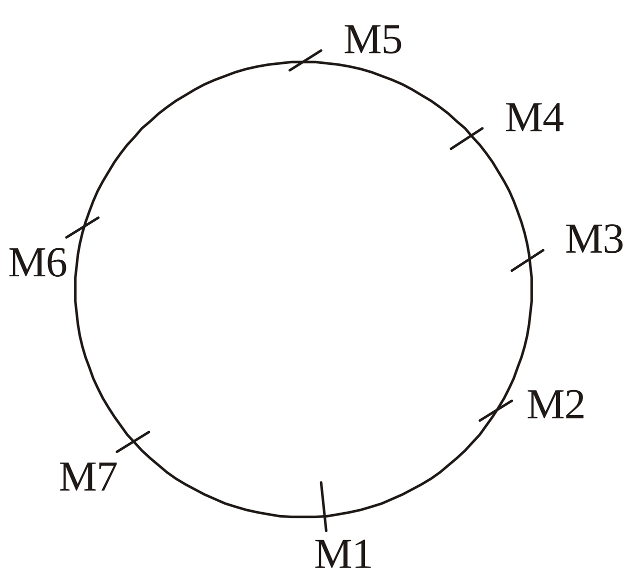
$$\begin{array}{l|l} m \quad n \quad 10 & from(i) \\ m-n \quad 6 & m \quad n \quad 10 \\ & 8 \quad n \quad 10 \\ 2m \quad 16 & n \quad 2 \\ (m \quad 8) & \end{array}$$

Circular Permutation

Theorem : The number of circular permutations of n different things is $\lfloor n-1$ ways.

Ques : In How many ways can 7 men and 7 women be seated at a round table so that no two women sit together ?

Solution :



The 7 men can be seated at the cirular table such that there is a vacant seat between every pair of men in $|\underline{7-1}$ ways i.e. $|\underline{6}$ ways. Now 7 vacant seat can be occupied by 7 women in $|\underline{7}$ ways.

Here the required number of seating arrangement $\quad |\underline{6} \quad |\underline{7}$ ways.

Ques : Find the number of arrangements of 12 things taken 5 at a time in which

(a) One particular thing is always included.

(b) 3 Particular things are always included.

Solution :

(a) Number of permutation of 12 objects taken 5 at a time (a particular object is always included.

$$^{5}P_{1} \quad ^{11}P_{4} \quad \frac{|\underline{5}}{|\underline{5-1}} \quad \frac{|\underline{11}}{|\underline{11-4}}$$

$$\frac{5 \quad |\underline{4}}{|\underline{4}} \quad \frac{|\underline{11}}{|\underline{7}}$$

$$\frac{5 \quad 11 \quad 10 \quad 9 \quad 8 \quad |\underline{7}}{|\underline{7}} \quad 39600$$

(b) Number of permutation of taking 3 particular object out of 12 objects.

$$^{5}P_{3} \quad ^{9}P_{2}$$

$$\frac{|\underline{5}}{|\underline{5-3}} \quad \frac{|\underline{9}}{|\underline{9-2}}$$

$$\frac{60 \quad 9 \quad 8 \quad |\underline{7}}{|\underline{7}} \quad 60 \quad 72 \quad 4320 \text{ Ans.}$$

In how many ways can the alphabets of the word BOKARO be arranged ?

Solution :

Total No. of letters in the word BOKARO = 6

Repeated letters 0 = 2 times

The total number of arrangement

$$\frac{\lfloor 6}{\lfloor 2} \quad \frac{6 \quad 5 \quad 4 \quad 3 \quad \lfloor 2}{\lfloor 2} = 360 \text{ ways.}$$

How many different words can be formed out of letters of the word COLLEGE.

Solution :

Total No. of letters = 7

Repeated letters L = 2 times and E = 2 times.

Total number of arrangement $\frac{\lfloor 7}{\lfloor 2 \quad \lfloor 2}$

$$\frac{7 \quad 6 \quad 5 \quad 4 \quad 3 \quad \lfloor 2}{\lfloor 2 \quad 2 \quad 1}$$

= 1260 ways.

In How many ways can the letters o the word 'STRANGE' be arranged so that the vowels letter may appear in the odd places ?

Solution :

$\textcircled{1}$ 2 $\textcircled{3}$ 4 $\textcircled{5}$ 6 $\textcircled{7}$

S T R A N G E

There are 7 letters.

There are four odd places (First, third, fifth and seventh) and two vowel letters (A and E)

Thus number of ways of filling odd places by vowels ${}^4P_2$ and rest five places are filled by ${}^5P_5$ways.

Total number of arrangement

$$({}^4P_2 \quad {}^5P_5) \text{ ways}$$

$$\frac{\underline{|4}}{\underline{|2}} \quad \frac{\underline{|5}}{\underline{|5-5}} \text{ ways}$$

$$\frac{\underline{|4}}{\underline{|2}} \quad \frac{\underline{|5}}{\underline{|0}} \quad \frac{4 \quad 3 \quad \underline{|2}}{\underline{|2}} \quad 5 \quad 4 \quad 2 \quad 1$$

= 1440 ways.

Prove that $\underline{|2n} \quad (2)^n \quad \underline{|n}\{1 \cdot 3 \cdot 5 \cdot .....(2n-1)\}$.

Proof : L.H.S = $\underline{|2n} \quad 2n(2n-1)(2n-2)(2n-3).....4 \quad 3 \quad 2 \quad 1$

$$\{2n(2n-2)(2n-4).....4 \quad 2)\} \quad \{(2n-1)(2n-3).....3 \quad 1\}$$

$$\{2 \quad n \quad 2(n-1) \quad 2(n-2).....2 \quad 2 \quad 2 \quad 1\}$$

$$\{(2n-1)(2n-3)....3 \quad 1\}$$

$$(2)^n\{n(n-1)(n-2).........2 \quad 1)\} \quad \{(2n-1)(2n-3).....3 \quad 1\}$$

$$(2)^n\underline{|n}\{(2n-1)(2n-3).....3 \quad 1\} \text{ = R. H.S.}$$

How many numbers between 100 and 1000 can be made with the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, ?

Solution :

All the numbers between 100 and 1000 will be of 3 digits.

i.e. $\boxed{101 \text{ to } 999}$

Total no. of numbers $\quad {}^9P_3 \quad \frac{\underline{|9}}{\underline{|9-3}}$

$$\frac{\underline{|9}}{\underline{|6}} \quad \frac{9 \quad 8 \quad 7 \quad \underline{|6}}{\underline{|6}}$$

$$72 \quad 7 \quad 504 \text{ Ans.}$$

How many numbers can be formed from the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits are at the odd places ?

Solution :

Total No. of digits = 7

$\textcircled{1}$ 2 $\textcircled{3}$ 4 $\textcircled{5}$ 6 $\textcircled{7}$
1, 2, 3, 4, 3, 2, 1

odd digits 1, 3, 3, 1

Even digits 2, 4, 2

Number of ways of filling odd digits at the odd places.

$$\frac{\underline{|4}}{\underline{|2} \quad \underline{|2}} \quad \frac{4 \quad 3 \quad \underline{|2}}{\underline{|2} \quad 2 \quad 1} = 6$$

Number of ways of filling even digits at even places.

$$\frac{\underline{|3}}{\underline{|2}} \quad \frac{3 \quad \underline{|2}}{\underline{|2}} \quad 3$$

Total number of ways = 6 × 3 = 18

Find the total number of numbers (a) Greater than 2000 (ii) Less than 3000 and (iii) Between 2000 and 3000 that can be formed with the digits 1, 2, 3, 4, 5. No digits being repeated in any number.

Solution :

(i) There are five digits

1, 2, 3, 4, 5

Thus, the number greater than 2000 is of either 4 digits or 5 digits.

Case I Four digits number.

2,3,4,5			

Thousand place can be filled in any ${}^4P_1$ ways and rest three place can be filled in Remaining 4 digits taken 3 at a time i.e. ${}^4P_3$ways.

Number of arrangement ${}^4P_1 \quad {}^4P_3$

$$\frac{\underline{|4}}{\underline{|4-1}} \quad \frac{\underline{|4}}{\underline{|4-3}}$$

$$\frac{\underline{|4}}{\underline{|3}} \quad \frac{\underline{|4}}{\underline{|1}}$$

$$\frac{4 \quad \underline{|3}}{\underline{|3}} \quad \frac{4 \quad 3 \quad 2 \quad \underline{|1}}{\underline{|1}} = 96$$

Case II Five digits number

$${}^5P_5 \text{ ways} = \frac{\underline{|5}}{\underline{|5-5}}$$

$$= \frac{\underline{|5}}{\underline{|0}} \quad \frac{5 \quad 4 \quad 3 \quad 2 \quad 1}{1} \quad 120$$

Total number of numbers which is greater than 2000

= 96 + 120 = 216 ways

(II) Less than 3000

Case I One digit number

$$^{5}P_{1} \quad \frac{\underline{|5}}{\underline{|5 \quad 1}} \quad \frac{5 \quad \underline{|4}}{\underline{|4}} \quad 5$$

Case II Two digits number

$$^{5}P_{2} \quad \frac{\underline{|5}}{\underline{|5 \quad 2}} \quad \frac{\underline{|5}}{\underline{|3}}$$

$$\frac{5 \quad 4 \quad \underline{|3}}{\underline{|3}} \quad 20$$

Case III Three digits number

$$^{5}P_{3} \quad \frac{\underline{|5}}{\underline{|5-3}} \quad \frac{\underline{|5}}{\underline{|2}}$$

$$\frac{5 \quad 4 \quad 3 \quad \underline{|2}}{\underline{|2}} \quad 60$$

Case IV Four digits number

1, 2,			

$$^{2}P_{1} \quad ^{4}P_{3} \quad \frac{\underline{|2}}{\underline{|2 \quad 1}} \quad \frac{\underline{|4}}{\underline{|4-3}}$$

$$\frac{\underline{|2}}{\underline{|1}} \quad \frac{\underline{|4}}{\underline{|1}}$$

$$\frac{2 \quad \underline{|1}}{\underline{|1}} \quad \frac{4 \quad 3 \quad 2 \quad \underline{|1}}{\underline{|1}} \quad 48$$

Total number of number in which is less than 3000

= 5 + 20 + 60 + 48 = 133 ways

(iii) Between 2000 and 3000 i.e four digits number

2			

$$1P_1 \quad 4P_3 \quad \frac{\lfloor 1}{\lfloor 1-1} \quad \frac{\lfloor 4}{\lfloor 4-3} \quad \frac{\lfloor 4}{\lfloor 1}$$

$$\frac{4 \quad 3 \quad 2 \quad \lfloor 1}{\lfloor 1} = 24 \text{ ways.}$$

Questions : How many arrangment can be formed with letters of the word DELHI. How many of them will begin with D and How many donot ? How many wil begin with D and end with I ? In How many word LH will be together ?

Solutions :

Thus there are 5 distinct letters. So the number of arrangement = $^5P_5$

$$\frac{\lfloor 5}{\lfloor 5-5} \quad \frac{\lfloor 5}{\lfloor 0} = 5\times4\times3\times2\times1 = 120 \text{ ways.}$$

(ii) Number of arrangement which begin with D

$$^4P_4 \text{ ways} = \frac{\lfloor 4}{\lfloor 4-4} \quad \frac{\lfloor 4}{\lfloor 0}$$

$$\frac{4 \quad 3 \quad 2 \quad 1}{1} \quad 24 \text{ ways.}$$

(iii) Number of arrangement which do not begin with D

= 120 – 24 = 96 ways.

(iv) Number of arrangement which begin with D and end with I.

D	E	L	H	I

$$={}^{3}P_{3} \quad \frac{\lfloor 3}{\lfloor 3-3} \quad \frac{\lfloor 3}{\lfloor 0} \quad \frac{3 \quad 2 \quad 1}{1} = 6 \text{ ways.}$$

(v) Number of arrangement when letters LH will be together

$${}^{4}P_{4} \quad {}^{2}P_{2} \text{ ways.}$$

DELHI $$\frac{\lfloor 4}{\lfloor 4-4} \quad \frac{\lfloor 2}{\lfloor 2-2}$$

$$\frac{4 \quad 3 \quad 2 \quad 1}{1} \quad \frac{2 \quad 1}{1}$$

$$= 48 \text{ ways.}$$

Combination

Combination deals with selection, Grouping or Choosing.

Each of the different groups or selection which can be made by taking some or all of a number of given things at a time. is called combination.

The symbol ${}^{n}C_{r}$ denotes the n different things selected r at a time.

i.e.

${}^{n}c_{r} \quad \frac{\lfloor n}{\lfloor r \lfloor n-r}$	$n \quad r$

where n = total number of given things

r = taken r at a time.

C= Combination

The following are some of the cases of combination.

(a) Formation of committee from given set of persons.

(b) Selection of Questions from Question Paper.

(c) Relation between permutation and combination.

$$\boxed{{}^nP_r \quad \underline{|r} \quad {}^nc_r}$$

Prove that

$$\boxed{{}^nc_r \quad {}^nc_{n-r}}$$

L.H.S $= {}^nc_r \quad \dfrac{\underline{|n}}{\underline{|n-r}\,\underline{|r}}$

R.H.S $= {}^nc_{n-r} \quad \dfrac{\underline{|n}}{\underline{|n-r}\;\underline{|n-(n-r)}}$

$$\frac{\underline{|n}}{\underline{|n-r}\;\underline{|n \quad (n-r)}}$$

$$\frac{\underline{|n}}{\underline{|n-r}\;\underline{|r}}$$

Prove that

$$\boxed{{}^nc_r \quad {}^nc_{r-1} \quad {}^{n\ \ 1}c_r}$$

L.H.S. $\quad {}^nc_r \quad {}^nc_{r-1}$

$$\frac{\underline{|n}}{\underline{|r}\;\underline{|n-r}} \qquad \frac{\underline{|n}}{\underline{|r \quad 1}\;\underline{|n-(r-1)}}$$

$$\frac{\underline{|n}}{r\underline{|r-1}\;\underline{|n-r}} \qquad \frac{\underline{|n}}{\underline{|r-1}\;\underline{|n-r \quad 1}}$$

$$\frac{\underline{|n}}{r\underline{|r-1}\,\underline{|n-r}} \quad \frac{\underline{|n}}{\underline{|r-1}\,(n-r \quad 1)\,\underline{|n-r}}$$

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r}} \quad \frac{1}{r} \quad \frac{1}{n-r \quad 1}$$

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r}} \quad \frac{n-r \quad 1 \quad r}{r(n-r \quad 1)}$$

$$\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r}} \quad \frac{n \quad 1}{r(n-r \quad 1)}$$

$$\frac{(n \quad 1)\,\underline{|n}}{r\underline{|r-1}\,(n-r \quad 1)\,\underline{|n \quad r}}$$

$^{n \; 1}c_r$ = R.H.S

In an Examination paper of Advanced Accountancy 10 Questions are set. In how many different ways can an examinee choose 7 Questions ?

Solutions :

Number of Questions (n) = 10

selected 7 questions at a time

Total number of combination = $^{10}C_7 \quad \frac{\underline{|10}}{\underline{|7}\,\underline{|10-7}}$

$$\frac{\underline{|10}}{\underline{|7}\,\underline{|3}} \quad \frac{10 \quad 9 \quad 8 \quad \underline{|7}}{\underline{|7} \quad 3 \quad 2 \quad 1}$$

= 120 ways.

If $^{n}c_4 \quad 5 \quad ^{n}P_3$ find the value of n.

Solutions :

$${}^{n}c_{4} \quad 5 \quad {}^{n}\mathrm{P}_{3}$$

$$\frac{\underline{|n}}{\underline{|4}\,\underline{|n-4}} \quad \frac{5 \quad \underline{|n}}{\underline{|n-3}}$$

$$\frac{1}{\underline{|4}\,\underline{|n-4}} \quad \frac{5 \quad 1}{(n-3)\underline{|n-4}}$$

$$n-3 \quad \underline{|4} \quad 5$$

$$n-3 \quad 4 \quad 3 \quad 2 \quad 1 \quad 5$$

n – 3 = 120

n = 123 Ans.

Question

$${}^{2n}c_{3} : {}^{n}c_{2} \quad 44:3$$

Find the value of n.

Solutions :

$$\frac{{}^{2n}c_{3}}{{}^{n}c_{2}} \quad \frac{44}{3}$$

$$\frac{\dfrac{\underline{|2n}}{\underline{|3}\,\underline{|2n-3}}}{\dfrac{\underline{|n}}{\underline{|2}\,\underline{|n-2}}} \quad \frac{44}{3}$$

$$\frac{\underline{|2n}}{\underline{|3}\,\underline{|2n-3}} \quad \frac{\underline{|2}\,\underline{|n-2}}{\underline{|n}} \quad \frac{44}{3}$$

$$\frac{2n(2n-1)(2n-2)\underline{|2n-3} \quad \underline{|2} \quad \underline{|n \quad 2}}{3 \quad \underline{|2} \quad \underline{|2n-3} \quad n(n \quad 1)\underline{|n-2}} \quad \frac{44}{3}$$

$$\frac{2n(2n-1) \quad 2(n-1)}{n(n-1)} \quad 44$$

4 × (2n –1) = 44

2n –1 = $\frac{44}{4}$

2n –1 = 11

2n = 12

$$\boxed{n \quad 6}$$

Question :

In how many ways can 4 students be selected out of 12 students if

(i) two particular students are not included at all (ii) Two particular students are included

Solutions :

Total number of students = 12

Number of students selected 4 at a time.

Total no. of selection $\quad {}^{12}c_4 \quad \frac{|\underline{12}}{|\underline{4}\,|\underline{12-4}}$

$$\frac{|\underline{12}}{|\underline{4}\,|\underline{8}} \quad \frac{12 \quad 11 \quad 10 \quad 9 \quad |\underline{8}}{4 \quad 3 \quad 2 \quad 1 \quad |\underline{8}}$$

= 45 ×11 = 495

(i) If 2 particular students are not taken then number of selection of 4 students out of remaining 10 students.

$${}^{10}c_4 \quad \frac{|\underline{10}}{|\underline{4}\,|\underline{10-4}} \quad \frac{|\underline{10}}{|\underline{4}\,|\underline{6}}$$

$$\frac{10 \quad 9 \quad 8 \quad 7 \quad \lfloor 6}{4 \quad 3 \quad 2 \quad 1 \quad \lfloor 6} \quad 210 \text{ Ans.}$$

(ii) If 2 particular students are always taken, then number of ways of selecting

$$^{2}c_{2} \quad ^{10}c_{2}$$

$$\frac{\lfloor 2}{\lfloor 2 \, \lfloor 2-2} \quad \frac{\lfloor 10}{\lfloor 2 \, \lfloor 10-2}$$

$$\frac{1}{\lfloor 0} \quad \frac{\lfloor 10}{\lfloor 2 \, \lfloor 8} \quad \frac{10 \quad 9 \quad \lfloor 8}{2 \quad 1 \quad \lfloor 8}$$

= 45 Ans.

Questions : From 6 boys and 4 girls, 5 are to be selected for admission for a particular course. In how many ways can this be done, if there must be exactly 2 girls.

Solutions :

Number of students selected for course

= 2girls and 3boys

There is condition that exactly 2 girls are taken

Number of selection of Boys = $^{6}c_{3}$

Number of selection of girls = $^{4}C_{2}$

Total number of combination

$$^{6}c_{3} \quad ^{4}c_{2}$$

$$\frac{\lfloor 6}{\lfloor 3 \, \lfloor 6-3} \quad \frac{\lfloor 4}{\lfloor 2 \, \lfloor 4-2}$$

$$\frac{\lfloor 6}{\lfloor 3 \, \lfloor 3} \quad \frac{\lfloor 4}{\lfloor 2 \quad \lfloor 2}$$

$$\frac{6\ \ 5\ \ 4\ \ \underline{|3}}{\underline{|3}\ \ 3\ \ 2\ \ 1}\ \frac{4\ \ 3\ \ \underline{|2}}{\underline{|2}\ \ 2\ \ 1}$$

= 120 ways

In how many ways 52 cards can be distributed to four players 13 each ?

Solution :

13 cards are distributed to first player in $^{52}C_{13}$ ways, Now 52–13 = 39 card left. So 13 card are distributed to second player, in $^{39}C_{13}$. Similarly for third player $^{26}C_{13}$ ways and for fourth player $^{13}C_{13}$ ways.

Total number of combination

$$^{52}c_{13}\ \ ^{39}c_{13}\ \ ^{26}c_{13}\ \ ^{13}c_{13}$$

$$\frac{\underline{|52}}{\underline{|13}\ \ \underline{|52-13}}\ \frac{\underline{|39}}{\underline{|13}\ \ \underline{|39-13}}\ \frac{\underline{|26}}{\underline{|13}\ \ \underline{|26-13}}\ \frac{\underline{|13}}{\underline{|13}\,\underline{|13-13}}$$

$$\frac{\underline{|52}}{\left(\underline{|13}\right)^{4}}$$ Ans.

Question : From 7 gentlement and 4 ladies a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady ?

Solutions :

Gentleman = 7
Ladies = 4
committee can be formed with 5 members

There is a condition at least one lady present.

Case I one lady and 4 gentleman

$$^{4}c_{1}\ \ ^{7}c_{4}$$

Case II 2 Ladies and 3 gentlement

$^4c_2 \quad ^7c_3$

Case III 3 Ladies and 2 gentlement

$^4c_3 \quad ^7c_2$

Case IV 4 Ladies and one gentlement

$^4c_4 \quad ^7c_1$

Thus total number ways

$^4c_1 \quad ^7c_4 \quad ^4c_2 \quad ^7c_3 \quad ^4c_3 \quad ^7c_2 \quad ^4c_4 \quad ^7c_1$

= 441 ways.