Nalanda Open University.

B.SC Part-2

Course : Physics (Hons)

Paper : 4

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Topic- Modern Physics (Compton Effect)

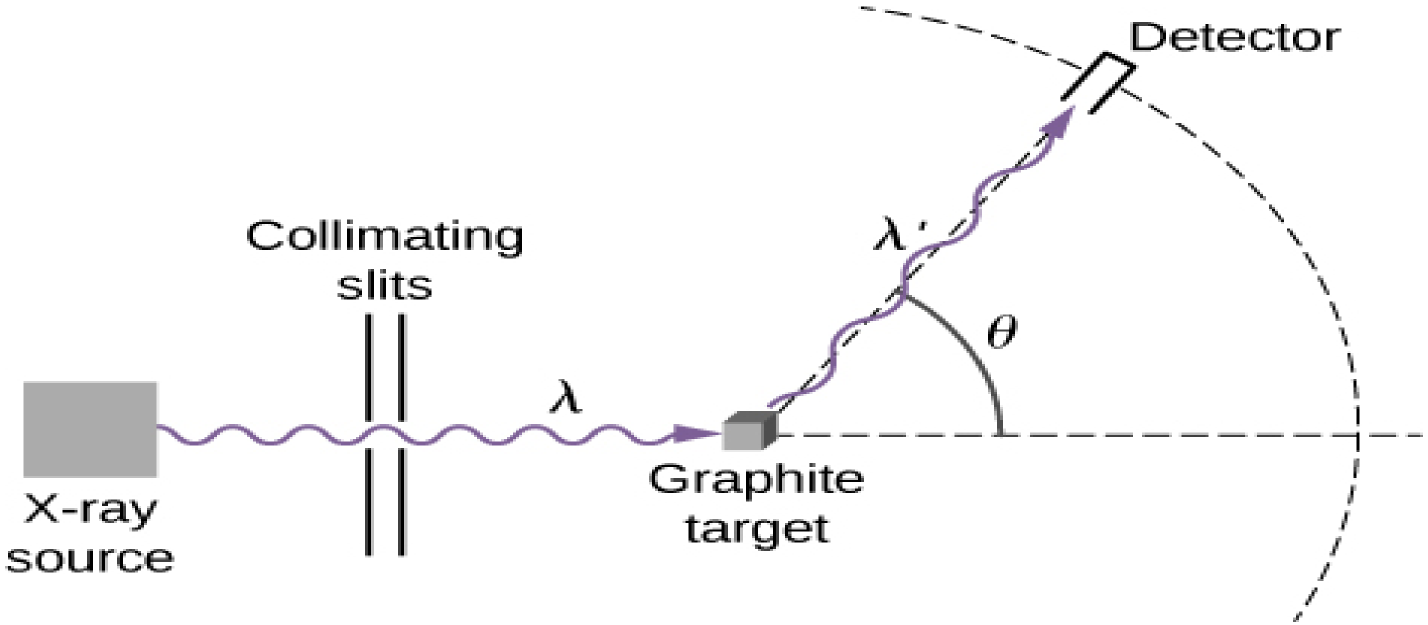
Compton Effect

The Compton effect is the term used for an unusual result observed when X-rays are scattered on some materials. By classical theory, when an electromagnetic wave is scattered off atoms, the wavelength of the scattered radiation is expected to be the same as the wavelength of the incident radiation. Contrary to this prediction of classical physics, observations show that when X-rays are scattered off some materials, such as graphite, the scattered X-rays have different wavelengths from the wavelength of the incident X-rays. This classically unexplainable phenomenon was studied experimentally by Arthur H. Compton and his collaborators, and Compton gave its explanation in 1923.

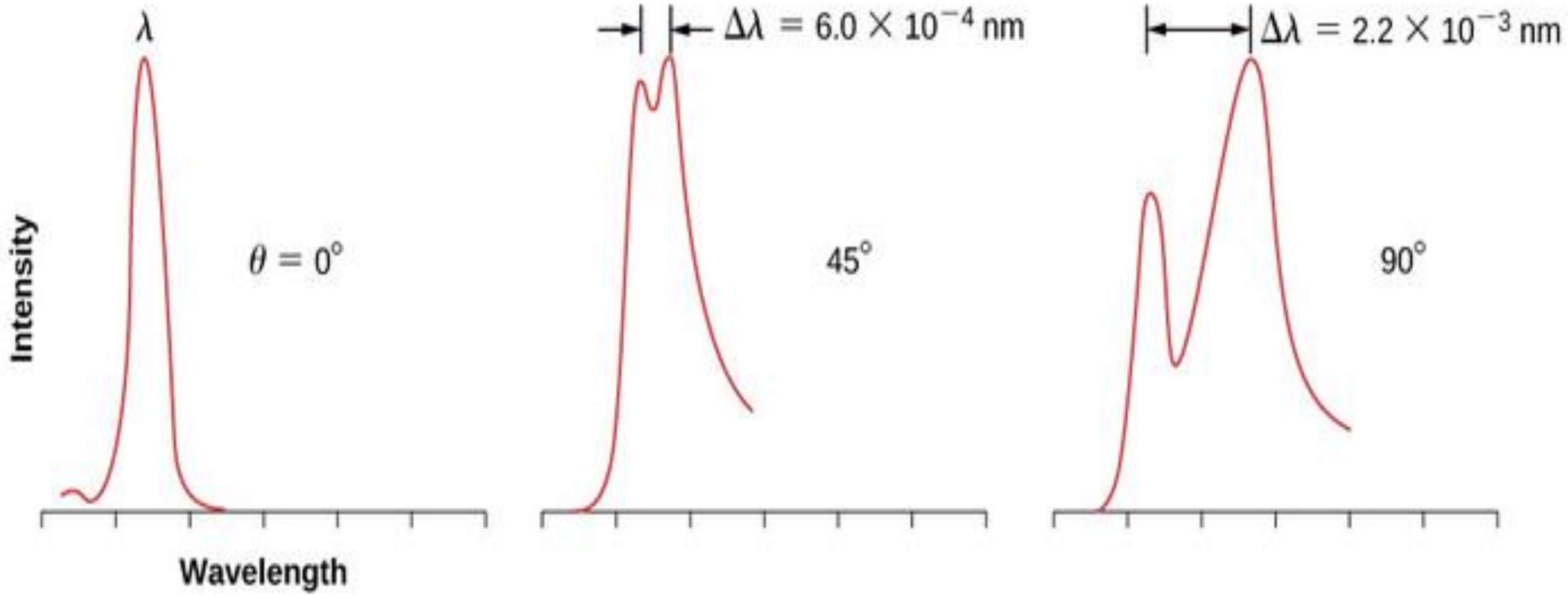
To explain the shift in wavelengths measured in the experiment, Compton used Einstein's idea of light as a particle. The Compton effect has a very important place in the history of physics because it shows that electromagnetic radiation cannot be explained as a purely wave phenomenon. The explanation of the Compton effect gave a convincing argument to the physics community that electromagnetic waves can indeed behave like a stream of photons, which placed the concept of a photon on firm ground.

The schematics of Compton's experimental setup are shown in (Figure) . The idea of the experiment is straightforward: Monochromatic X-rays with wavelength λ are incident on a sample of graphite (the "target"), where they interact with atoms inside the sample; they later emerge as scattered X-rays with wavelength $\lambda\prime$. A detector placed behind the target can measure the intensity of radiation scattered in any direction θ with respect to the direction of the incident X-ray beam. This scattering angle, θ, is the angle between the direction of the scattered beam and the direction of the incident beam. In this experiment, we know the intensity and the wavelength λ of the incoming (incident) beam; and for a given scattering angle θ, we measure the intensity and the wavelength $\lambda\prime$ of the outgoing (scattered) beam. Typical results of these measurements are shown in (Figure) , where the *x*-axis is the wavelength of the scattered X-rays and the *y*-axis is the intensity of the scattered X-rays, measured for different scattering angles (indicated on the graphs). For all scattering angles (except for $\theta = 0$), we measure two intensity peaks. One peak is located at the wavelength λ, which is the wavelength of the incident beam. The other peak is located at some other wavelength, $\lambda\prime$. The two peaks are separated by λ, which depends on the scattering angle θ of the outgoing beam (in the direction of observation). The separation λ is called the Compton shift.

Experimental setup for studying Compton scattering.



Experimental data show the Compton effect for X-rays scattering off graphite at various angles: The intensity of the scattered beam has two peaks. One peak appears at the wavelength λ of the incident radiation and the second peak appears at wavelength $\lambda\prime$. The separation λ between the peaks depends on the scattering angle θ, which is the angular position of the detector in (Figure) . The experimental data in this figure are plotted in arbitrary units so that the height of the profile reflects the intensity of the scattered beam above background noise.


λ
Δλ = 6.0 × 10⁻⁴ nm
Δλ = 2.2 × 10⁻³ nm
Intensity
θ = 0°
45°
90°
Wavelength

Compton and Debye regarded the incident x-ray beam as a collection of photons, and not as waves, each of energy $E_0 = h\nu_0 = hc/\lambda_0$. They suggested that λ_1 could be attributed to scattering of x-ray photons from loosely bound electrons in the atom of the target, where they loose some of its energy in the inelastic collision, $E_1 < E$. Therefore, their frequency is reduced implying larger wavelength $\lambda_1 = c/\nu_1 = hc/E_1$. Since the electrons participating in the scattering process are treated almost free and initially stationary (binding energy of the electrons are small compared to the energy of the x-ray photons) and does not involve entire atoms, this kind of explains why $\Delta\lambda$ is independent of the material of the scatterer.

To calculate the Compton shift, let a photon of total energy E_0 and momentum p_0 is incident on a stationary electron of rest mass energy m_0c^2,

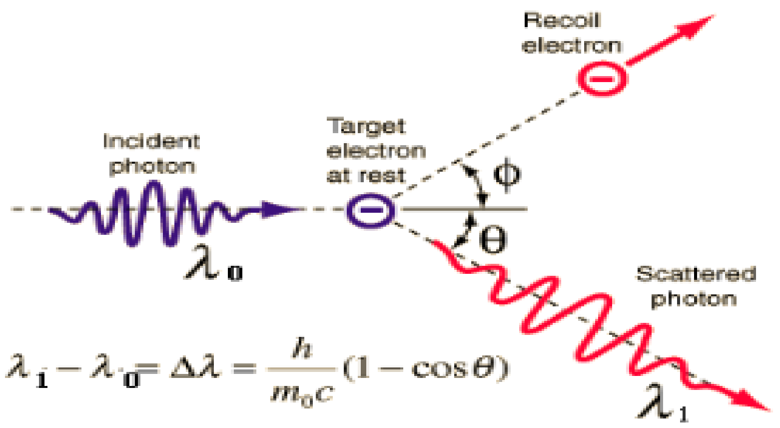
$$E_0 = h\,\nu_0 = \frac{hc}{\lambda_0} \quad \text{and} \quad p_0 = \frac{E_0}{c} = \frac{h}{\lambda_0}. \tag{1}$$

After the collision, the photon is scattered at an angle θ and moves off with total energy E_1 and momentum p_1,

$$E_1 = h\,\nu_1 = \frac{hc}{\lambda_1} \quad \text{and} \quad p_1 = \frac{E_1}{c} = \frac{h}{\lambda_1}. \tag{2}$$

and electron recoils at an angle ϕ with kinetic energy K, total energy E and momentum p,

$$E^2 = p^2c^2 + m_0^2c^4 \quad \text{and} \quad K = E - m_0\,c^2. \tag{3}$$



Momentum conservation leads to,

$$\begin{aligned} p_0 &= p_1\,\cos\theta + p\,\cos\phi \\ 0 &= p_1\,\sin\theta - p\,\sin\phi. \end{aligned}$$

Squaring and adding the above two equations, we get

$$p^2 = p_0^2 + p_1^2 - 2p_0p_1\,\cos\theta. \tag{4}$$

From conservation of energy in the collision, it follows that

$$E_0 + m_0c^2 = E_1 + E \;\Rightarrow\; E = (E_0 - E_1) + m_0c^2 \tag{5}$$

and using equations (21) and (22), we obtain

$$\left(p^2c^2 + m_0^2c^4\right)^{1/2} = c(p_0 - p_1) + m_0c^2 \tag{6}$$

which upon squaring gives us,

$$p^2 = (p_0 - p_1)^2 + 2m_0c(p_0 - p_1). \tag{7}$$

Comparing equations (4) and (7) , we have

$$(p_0 - p_1)^2 + 2m_0c(p_0 - p_1) = p_0^2 + p_1^2 - 2p_0p_1 \cos\theta$$

which reduces to

$$\frac{1}{p_1} - \frac{1}{p_0} = \frac{1}{m_0c}(1 - \cos\theta). \tag{8}$$

Multiplying through by h and applying (1) and (2) we obtain the Compton equation

$$\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_c(1 - \cos\theta) \tag{9}$$

where, λ_c is the Compton wavelength defined as,

$$\lambda_c \equiv \frac{h}{m_0c} = 0.0243\mathring{A}. \tag{10}$$

A few more lines of calculation gives us the relation between scattering and recoil angle and kinetic energy of the recoiled electron (using $\alpha = \lambda_c\nu_0/c = \lambda_c/\lambda_0$),

$$\cot\phi = (1 + \alpha)\tan\frac{\theta}{2}, \tag{11}$$

$$K = h\nu_0\frac{\alpha(\cos\theta - 1)}{1 + \alpha(\cos\theta - 1)}. \tag{12}$$

To explain the presence of peak at unchanged photon wavelength λ_0, we observed that if the electron involved in scattering are particularly strongly bound to the atom in the target then the whole atom recoils. Therefore, the electron rest mass m_0 in Compton equation (9) , has to be replaced by mass of the atom $M \gg m_0$ and hence the Compton shift becomes way too small, $\Delta\lambda \sim 1/M$.

Summary

- In the Compton effect, X-rays scattered off some materials have different wavelengths than the wavelength of the incident X-rays. This phenomenon does not have a classical explanation.
- The Compton effect is explained by assuming that radiation consists of photons that collide with weakly bound electrons in the target material. Both electron and photon are treated as relativistic particles. Conservation laws of the total energy and of momentum are obeyed in collisions.
- Treating the photon as a particle with momentum that can be transferred to an electron leads to a theoretical Compton shift that agrees with the wavelength shift measured in the experiment. This provides evidence that radiation consists of photons.
- Compton scattering is an inelastic scattering, in which scattered radiation has a longer wavelength than that of incident radiation.