M C A

PART - I

SUBJECT
DISCRETE MATHEMATICS

PAPER - III

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PROPOSITIONAL CALCULUS

Logical Equivalence

Precedence Rule

Logical Quantifier

Questions for Exercise

Logical Equivalence

Two Propositions (Simple or compound) are said to be logically equivalent if they have the same truth value If "p" is logically equivalent to "q"

Then we write p = q

Example : Show that

(p q) (p q)

Proof :

p	q	p	p q	p q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From the above table the truth value of (p q) is same as the truth value of (p q)

Hence (p q) (p q)

1.9.2 The following properties follow from the definition of logical equivalence.

(a) p p

(b) if p q and q r then p r

(c) p q q p

i.e. An implication and its contrapositive are logically equivalent.

(d) q p p q

i.e. The converse of an implication and the inverse are logically equivalent.

1.9.3 Theorem :

The Negation of negation of a Proposition is logically equivalent to the given proposition i.e. (p) p

Proof :

p	p	(p)
T	F	T
F	T	F

1.9.4 The Negation of the Conjunction of two Proposition is logically equivalent to the disjunction of their negation.

i.e. (p q) p q

Proof :

p	q	p	q	p q	p q	(p q)
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

1.9.5 The Negation of the disjunction of two Propositions is logically equiavalent to the conjunction of their negation.

i.e. (p q) p q

Truth Table

p	q	p	q	p q	p q	(p q)
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

1.9.6 The negation of an implication is logically equivalent to the conjunction of its hypothesis and the negation of the conclusion.

i.e. (p q) p (q)

Proof :

p	q	q	p q	(p q)	p q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

1.9.7 The following are some standard logically equivalent statements.

(1) Idempotent Laws

p p p

p p p

(2) Commutative Law

p q q p

p q q p

(3) Associative Law

(p q) r p (q r)

(p q) r p (q r)

(4) Distributive Laws

p (q r) (p q) (p r)

p (q r) (p q) (p r)

(5) Demorgan's Laws

(p q) p q

$(p \quad q) \quad p \quad q$

Precedence Rule

When we are dealing with operations on number, you would have realized the need for applying the BODMAS rule. According to this rule, when calculating the value an arithmetic expression, we first calculate the value of the Bracketed Portion, then apply of, Division, Multiplication, Addition and substraction, in this order while calculating the truth value of compound statements involving more than one connective we have a similar connection while tells us which connective to apply first.

Suppose we did not have an order of preference, and want to find the truth of, say $p \quad q$. Some of us may consider the value of $(\quad p) \quad q$ and some may consider $(p \quad q)$. The truth values can be different in these cases. For Example of p and q are both true, then $(\quad p) \quad q$ is true but $(p \quad q)$ is false

The rule of Precedence : The order of Preference in which the connectives are applied in a formula of propositions that has no brackets is.

(i)

(ii)

(iii) and

(iv) and

Here the "Inclusive or" and "Exclusive or" are both third in the order of preference

As for Example :

In $p \quad q \quad p$, we first apply v and then . The same applies to the implication and the Biconditional which are both fourth in the order of Preference.

1.10.2 Write down the truth table of $(p \quad q) \quad r \quad r \quad q$

Proof : Truth Table

p	q	r	r	q r	r q	p q r	p q r
T	T	T	F	F	F	F	T
T	T	F	T	T	T	T	T
T	F	T	F	F	T	F	F
T	F	F	T	F	F	F	F
F	T	T	F	F	F	T	F
F	T	F	T	T	T	T	T
F	F	T	F	F	T	T	T
F	F	F	T	F	F	T	F

Logical Quentifiers

Quantifiers are used for generality or exception in a propositional formulae or function. Main there are two types of quantifiers first is universal quatifiers and second is existential quantifiers.

Universal Quantifier

The Expression —, xp(x), is the universal quantification of P(x). If we translate this into the English language, the expression is : 'P(x) holds for all x', — stands for the universal quantifier followed by the predicate P(x), that conveys the meaning that P(x) is true for every object x in the universal. For Example, the statement 'All cars have wheels' can be expressed in a Propositional form, — xP(x), where

P(x) is the Predicate telling : x has wheels

The universe of discourse is population of cars.

Existential Quantifier

The expression, $xp(x)$, is a symbol used as existential quantifier of predicate denoted as P(x). Its English equivalent is : There exists an x such that P(x) is true'. is called the existential quantifier, which means at least one object. P(x) that follows this conveys that P(x) is true for at least one object x of the universe. For example, 'Some loves you' can be written as xp(x) in Propositional form.

The statement 'for all x, P(x)' is represented by — xP(x) and has a truth value. For Example, if P(x) is given as 'x + 2 = 7' and the domain is the set of integers then — xP(x) is true. But if a propositional, function, Q(x) conveys '$(x+1')^2 = x^2 + 2x + 1$' then — xQ(x) is true the symbol — is known as universal quantifier.

In case where a predicate uses multiple variable, seversal quantifiers can be used at the same time. For example English equivalent of — x — y zP(x,y,z), is 'for all x and all y there is some Z such that P(x,y,z) is true. There are many branches of logic which use quantifiers other than these two. In quantified Proporsition, the order of quantifiers are important. Thus — x yP(x,y) has a meaning difference than y — xp(x,y).

For Example If x and y stands for two persons such that P(x, y) means 'x is married to y, — x $yP(x_1y)$ conveys the meaning that there is at least one to whom every body is married. y — xP(x,y) conveys the meaning that every body else is married to some one.

Universal Quantifier and Connective AND

If it is possible to list every element and so the universal quantification —xP(x) $P(x_1)$ $P(x_2)$ $P(x_3)$ .... $P(x_n)$ For Example —xP(x) if it is given that only 3 cars are there (C_1, C_2, and C_3) then in statement it can be translated as $P(C_1)$ $P(C_2)$ $P(C_3)$

Existential Quantifier and Commutive OR

If it is possible to list every element that belongs to universe of discourse then quantification using existential quantifier $xP(x)$ is equivalent to the disjunction $P(x_1)$ $P(x_2)$ $P(x_3)$ .... $P(x_n)$.

For Example in $xP(x)$, if it is given that only 4 living creatures are there namely, Rakesh, Mukesh, Rajesh and Amaresh then the statement can be written as P(Rakesh) P(Mukesh) P(Rajesh) P(Amaresh)

1.13 Questions for Exercise

1. Define the statement or proposition.

2. Which of the following sentences are statements ? What are the reasons for your answer ?

   (a) the sun rises in the west

   (b) How far is Delhi from here ?

   (c) Smoking is injurious to health.

   (d) There is no rain without clouds.

   (e) What is a beautiful day !

   (f) She is an engineering Graduates.

   (g) Mathematics is fun.

3. Define the following with truth table.

   (a) Conjunction (b) Disjunction (c) conditional or Implication

   (d) Bi-conditional (f) Negation (g) converse (h) Inverse (i) Contrapositive of an Implication

4. Write the t ruth table $P \quad (q \quad r)$

5. Define the Tautology and Prove that $p \quad (p \quad q)$ is a tautology

6. Define contradiction and Prove that $(p \quad q) \quad p$ is a contradiction.

7. Prove that $r \quad (\quad r \quad s)$ is neither a tautology nor a contradiction.

8. Prove that the Negation of the conjunction of two Propositions is logically equivalent to the disjunction of their negation.

i.e. $(p \quad q) \quad p \quad q$

9. Prove that the negation of the disjunction of two statements is logically equivalent to the conjunction of their negation.

i.e. $(p \quad q) \quad p \quad q$

10. Prove that

$[(p \quad q) \quad (q \quad r)] \quad (p \quad q)]$ is a tautology.

11. Prove that $(p \quad q) \quad (p \quad q)$ is a tautology.