

**Nalanda Open University**  
**Department of Economics**

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Course – MA Economics (Part –I)  
Paper – III (Quantitative Methods)  
Topic – Correlation

Correlation Coefficient

(i) Karl Pearson Method 
$$r (X, Y) = \frac{\text{Cov} (X,Y)}{(\text{SD of X}) (\text{SD of Y})}$$

(ii) Product Moment Correlation

$$r (X, Y) = \frac{N\sum XY - \sum X\sum Y}{(N\sum X^2 - (\sum X)^2)^{1/2} ((N\sum Y^2 - (\sum Y)^2)^{1/2}}$$

(iii) Rank Correlation 
$$r (X, Y) = 1 - \frac{6\sum D^2}{n (n^2 - 1)}$$

Question - Find the correlation coefficient of the following with

(i) Product Moment Method and (ii) Karl Pearson Method

X	23	27	28	28	29	30	31	33	35	36
Y	18	20	22	27	21	29	27	29	28	29

Solution: - (i) Product Moment Method

X	Y	X <sup>2</sup>	Y <sup>2</sup>	XY
23	18	529	324	414
27	20	729	400	540
28	22	784	484	616
28	27	784	729	756
29	21	841	441	609
30	29	900	841	870
31	27	961	729	837
33	29	1089	841	957
35	28	1225	784	980
36	29	1296	841	1044
$\sum X = 300$	$\sum Y = 250$	$\sum X^2 = 9138$	$\sum Y^2 = 6414$	$\sum XY = 7623$

Product Moment Correlation  $r(X, Y) = \frac{N\sum XY - \sum X \sum Y}{(N\sum X^2 - (\sum X)^2)^{1/2} ((N\sum Y^2 - (\sum Y)^2)^{1/2}}$

$$N\sum XY - \sum X \sum Y = 10 * 7623 - 300 * 250$$

$$= 1230$$

$$\sqrt{N\sum X^2 - (\sum X)^2} = [10 * 9138 - (300 * 300)]^{1/2}$$

$$= [91380 - 90000]^{1/2}$$

$$= 37.15$$

$$\sqrt{N\sum Y^2 - (\sum Y)^2} = [10 * 6414 - (250 * 250)]^{1/2}$$

$$= [64140 - 62500]^{1/2}$$

$$= 40.5$$

$$\text{Now, } r(X,Y) = \frac{1230}{37.15 \times 40.5} = + 0.82 \text{ Answer}$$

(ii) Karl Pearson Method

X	Y	X – Mean of X	Y – Mean of Y	(X – Mean of X) <sup>2</sup>	(Y – Mean of Y) <sup>2</sup>	(X – Mean of X) * (Y – Mean of Y)
23	18	-7	-7	49	49	49
27	20	-3	-5	9	25	15
28	22	-2	-3	4	9	6
28	27	-2	2	4	4	-4
29	21	-1	-4	1	16	4
30	29	0	4	0	16	0
31	27	1	2	1	4	2
33	29	3	4	9	16	12
35	28	5	3	25	9	15
36	29	6	4	36	16	24
Mean of X = 30	Mean of Y = 25	$\sum(X - \text{Mean of X}) = \text{Zero}$ (Deviation from Mean is always Zero)	$\sum(Y - \text{Mean of Y}) = \text{Zero}$ (Deviation from Mean is always Zero)	$\sum(X - \text{Mean of X})^2 = 138$	$\sum(Y - \text{Mean of Y})^2 = 164$	$\sum (X - \text{Mean of X}) * (Y - \text{Mean of Y}) = 123$

The covariance is:

$$\text{Cov}(X,Y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

Standard Deviation of X:

$$\text{SD of X} = \sqrt{\frac{\sum (X - \text{Mean of X}) * (X - \text{Mean of X})}{n}}$$

Standard Deviation of Y:

$$\text{SD of Y} = \sqrt{\frac{\sum (Y - \text{Mean of Y}) * (Y - \text{Mean of Y})}{n}}$$

The covariance is:

$$\text{Cov}(X,Y) = \frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

$$\text{Cov}(X,Y) = \frac{\sum (X - \text{Mean of X}) * (Y - \text{Mean of Y})}{n}$$

$$= \frac{123}{10}$$

$$= 12.3$$

Standard Deviation of X:

$$\text{SD of X} = \sqrt{\frac{\sum (X - \text{Mean of X}) * (X - \text{Mean of X})}{n}}$$

$$= \sqrt{\frac{138}{10}}$$

$$= \sqrt{13.8}$$

$$= 3.7148$$

Standard Deviation of Y:

$$\text{SD of Y} = \sqrt{\frac{\sum(Y - \text{Mean of Y}) * (Y - \text{Mean of Y})}{n}}$$

$$= \sqrt{\frac{164}{10}}$$

$$= \sqrt{16.4}$$

$$= 4.0497$$

Karl Pearson Method

$$r(X, Y) = \frac{\text{Cov}(X, Y)}{(\text{SD of X})(\text{SD of Y})}$$

$$= \frac{12.3}{3.7148 * 4.0497}$$

$$= 0.8176$$

$$= + 0.82 \text{ (approx.)}$$

Question – Find the Correlation Coefficient of the following with Rank Method.

X	68	66	78	50	67	80	75	40	55	64
Y	62	58	67	45	81	60	68	48	50	70

Solution: -

Solving the problem by ORDERING in ASCENDING way

X	Y	R <sub>X</sub>	R <sub>Y</sub>	D= (R <sub>X</sub> - R <sub>Y</sub> )	D <sup>2</sup>
68	62	7	6	1	1
66	58	5	4	1	1
78	67	9	7	2	4
50	45	2	1	1	1
67	81	6	10	-4	16
80	60	10	5	5	25
75	68	8	8	0	0
40	48	1	2	-1	1
55	50	3	3	0	0
64	70	4	9	-5	25

Here  $\sum D^2 = 74$  and  $n = 10$

$$\begin{aligned}
 \text{Rank Correlation } r(X, Y) &= 1 - \frac{6\sum D^2}{n(n^2-1)} \\
 &= 1 - \frac{6*74}{10(10*10-1)} \\
 &= 1 - \frac{444}{10*99} \\
 &= \frac{990-444}{990} \\
 &= +0.55
 \end{aligned}$$

Solving the problem by ORDERING in DECENDING way

X	Y	R <sub>X</sub>	R <sub>Y</sub>	D= (R <sub>X</sub> - R <sub>Y</sub> )	D <sup>2</sup>
68	62	4	5	1	1
66	58	6	7	1	1
78	67	2	4	-2	4
50	45	9	10	-1	1
67	81	5	1	4	16
80	60	1	6	-5	25
75	68	3	3	0	0
40	48	10	9	1	1
55	50	8	8	0	0
64	70	7	2	5	25

Here  $\sum D^2 = 74$  and  $n = 10$

$$\begin{aligned}
 \text{Rank Correlation } r(X, Y) &= 1 - \frac{6\sum D^2}{n(n^2-1)} \\
 &= 1 - \frac{6*74}{10(10*10-1)} \\
 &= + 0.55
 \end{aligned}$$