Nalanda Open University

Annual Examination - 2016

B.Sc. Mathematics (Honours), Part-I

Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) Define the cartesian product of two non-empty sets A and B and if $\{Xi\}_{i \in I}$ and $\{Yi\}_{i \in I}$ be families of subsets of a set X indexed by the same set I then show that

$$\left(\prod_{i \in I} Xi\right) \cup \left(\prod_{i \in I} Yi\right) \subseteq \prod_{i \in I} (Xi \cup Yi) \text{ and } \left(\prod_{i \in I} Xi\right) \cap \left(\prod_{i \in I} Yi\right) = \prod_{i \in I} (Xi \cap Yi)$$

(b) Define a relation on a non-empty set A if R_1 and R_2 are any two equivalence relations on a Set A then prove that $R_1 \cap R_2$ is also an equivalence relation on A.

2. (a) Show that for any non-empty Set X, $(P(X), \subseteq)$ is a partially ordered set and it is not totally ordered if it has more that one element, where P(X) is the power Set of X.

(b) If $f: X \to Y$ and $A \subseteq Y$, $B \subseteq Y$ then show that
$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$

Group - B

3. (a) Define a Lattice and a complete Lattice. Give an example of a complete Lattice.

(b) In a Lattice prove that $(x \vee y) \wedge x = x$ and $(x \wedge y) \vee x = x$

4. (a) If $f: X \to Y$, $A \subseteq X$, $B \subseteq X$ than show that $f(A \cap B) \subseteq f(A) \cap f(B)$ and equality will hold if f is one to one.

(b) Show that a countable union of countable set is countable.

5. (a) Show that the set R of all real numbers is uncountable.

(b) Prove that every infinite set has a denumerable subset.

Group - C

6. If the matrices A and B are conformal for the product AB and B and C are conformal for the product BC then prove that (AB)C = A(BC) where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ -3 & -1 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix} \; C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$$

7. (a) Define transpose of a matrix and prove that $(AB)' = B'A'$ where A and B are matrics for the conformal product.

(b) Prove that a square matrix can be expressed uniquely as a sum of a symmetric and a skew-symmetric matrix.

8. (a) If A be a n-rowed square matrix then prove that
(adj A) A = A(adj A) = $|A| I_n$ where $|A|$ is the determinant of A and I_n is the unit matrix of order n.

(b) Find the invurse of the matrix

$$A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

Group - D

9. (a) Show that a group of three elements is necessarily abelian.

   (b) Prove that the order of a cyclic group is equal to the order of its generator.

10. (a) Prove that the intersection of two subgroups of a group is the subgroup of that group.

    (b) State and prove Lagrange's Theorem.

Group - E

11. (a) Solve the equation $3x^3 - 11x^2 + 8x + 4 = 0$ given that there are two equal roots.

    (b) Solve $x^3 + 6x^2 + 9x + 4 = 0$ by Cardon's method.

12. (a) If $x_r = Cos\frac{\pi}{2r} + iSin\frac{\pi}{2r}$ then prove that $x_1.x_2.x_3 ....to\infty = 1$

    (b) If $i^{i^{i^{........\infty}}} = A + iB$ then prove that $\tan\left(\frac{\pi A}{2}\right) = \frac{B}{A}$ and $A^2 + B^2 = e^{-n\pi}$

Nalanda Open University

Annual Examination - 2016

B.Sc. Mathematics (Honours), Part-I

Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting at least one from each group. All questions carry equal marks.*

Group - A

1. (a) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ then show that :
   $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$
   (b) If $y = (\text{Sin}^{-1}x)^2$, then prove that
   $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$
2. (a) Deduce Maclaurin's series.
   (b) Apply Maclaurin's series to expand $e^{\text{Sin}\,x}$ as far as the term involving x^4.
3. (a) Find the Lagrange's form of remainder after n terms in the expansion of $log(1 + x)$.
   (b) Prove that θ which occurs in Lagrange's form of remainder i.e. $\frac{h^n}{n!} f^n(a+\theta h)$ tends to the limit $\frac{1}{n+1}$ when $h \to 0$ provided $f^{n+1}(x)$ is continuous at a and $f^{n+1}(a) \neq 0$.

Group - B

4. (a) If x Cosα + y Sinα = p touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$ then show that
   $(a Cos\,\alpha)^{\frac{m}{m-1}} + (b Sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$
   (b) Find the derivative of the arc length of the curve $y = a \log Sec \frac{x}{a}$
5. (a) Show that the curves $r = a\,(1 + Cos\,\theta)$ and $r = b(1 - Cos\,\theta)$ cut each other orthogonally.
   (a) State and prove Euler's theorem on Homogeneous function of three variables.
6. (a) Prove that for radius of curvature at the point (r, θ) for the curve $r^n = a^n\, Cos\, n\theta$
   (b) Find the asymptotes of the curve $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 1$
7. Evaluate any two of the following:
   (i) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (ii) $\int \frac{dx}{\sqrt{2+x-3x^2}}$ (iii) $\int \sqrt{x^2+a^2}\; dx$

Group – C

8. (a) Prove that the chord of contact of tangents drawn from an external point to the conic $ax^2 + 2hxy + by^2 = 1$ subtends a right angle at the centre if the point lies on the conic
   $(a^2 + h^2)x^2 + 2h(a + b)xy + (h^2 + b^2)y^2 = a + b$
   (b) Find the pair of tangents from (1,1) to $2x^2+y^2-4x+2y+2=0$
9. (a) Deduce the equation of the conic in the form $\frac{l}{r} = 1 + e\,Cos\,\theta$
   (b) If α is the vectorial angle of one extremity of a focal chord of the ellipse $\frac{l}{r} = 1 + e\,Cos\,\theta$.
   Then prove that the angle between the tangents at its both extremities is $\tan^{-1} \frac{2\,e\,Sin\,\alpha}{1-e^2}$

Group – D

10. (a) Find the equations of the spheres which passing through four given points.
    (b) If the tangent to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts on the co-ordinate axes a,b,c then prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$
11. (a) Find the equation of the conic whose vertex is (α, β, γ) and the guiding curve is the cone.
    $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, z = 0.$
    (b) Prove that the sum of the squares of any three conjugate semi-diameters of an ellipsoid is constant.

Nalanda Open University
Annual Examination - 2016
B.Sc. Mathematics (Subsidiary), Part-I
Paper-I

Time: 3.00 Hrs. **Full Marks: 80**

*Answer any **Five** questions, selecting atleast **One** question from each group.*
All questions carry equals marks.

Group - A

1. (a) Define the cartesian product of two non-empty sets A and B and prove that A x (B – C) = A x B – A x C for three non-empty sets A, B and C.
   (b) What do you mean by a relation. Define reflexive, symmetric and transitive reltations giving one example of each.
2. (a) State and prove fundamental theorem of an equivalence relation.
   (b) If $f : X \to Y$ and $A \subseteq X$, $B \subseteq X$ then prove that $f(A \cup B) = f(A) \cup f(B)$.
3. Prove that the set R of all real numbers is uncountable.
4. (a) Show that a group of three elements is necessarily abelian.
   (b) Define cyclic group and prove that the order of a cyclic group is equal to the order of its generator.

Group - B

5. (a) State and prove De=Moivre's Theorem.
   (b) Find the three roots of *(1 + i)*
6. (a) Prove that $(i)^i = e^{-(4n+1)\frac{\pi}{2}}$
   (b) Reduce $(\alpha + i\beta)^{x+iy}$ in the form A + *i*B.

Group - C

7. (a) Prove that the limit of a sequence is unique.
   (b) Prove that a monotonically increasing sequence tends to its least upper bound.
8. (a) Find the equation of the circle which cuts orthogonally each of the three circles $x^2 + y^2 + 2x + 17y + 4 = 0$, $x^2 + y^2 + 7x + 6y + 11 = 0$ and $x^2 + y^2 - x + 22y + 3 = 0$.
   (b) Prove that from a given point $P(x_1, y_1)$ in general three normals can be drawn to the parabola $y^2=4ax$.

Group - D

9. (a) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ then show that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$
   (b) Using Maclaurin's series expand $e^{\sin x}$ as far as the term involving x^4.
10. (a) If $u = Sin^{-1} \dfrac{x^2 + y^2}{x + y}$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = n \tan u$
    (b) Evaluate the limit : $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.
11. (a) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$
    (b) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$ or $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a} \ \ \vec{b} \ \ \vec{c}]$

Nalanda Open University
Annual Examination - 2016
B.Sc. Mathematics (Honours), Part-II
Paper-III

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) If L and L^1 are two cuts theie show that only one of the following three relations hold.
$$L < L^1, L = L^1, L > L^1$$
   (b) State and prove order completeness theorem for real number system.
2. (a) State and prove Fundamental theorem of classical analysis.
   (b) Show that the intersection of two open sets is open in R.
3. (a) State and prove Bolzano-Weierstrass theorem.
   (b) Find C of Lagrange's Mean Value Theorem for
   $f(x) = (x-1)(x-2)(x-3);\ a = 0,\ b = 4.$

Group-B

4. (a) Prove that a monotonically decreasing sequence which is bounded tends to its greatest lower bound.
   (b) Show that the sequence (x_n) defind by
   $X_n = (\sqrt{n+1} - \sqrt{n})\ \forall\, n \in N$ is convergent and find its limit.
5. (a) State and prove Cauchy's General principle of convergence of a sequence.
   (b) Prove that the series $\sum \frac{1}{n^p}$ is convergent for p > 1 and divergent for p ≤ 1.
6. (a) State and prove Logarithmic ratio test.
   (b) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log^n}}$
7. (a) State and prove Abel's test.
   (b) Show that the series $\sum_{n=1}^{\infty} \frac{Cos\, nx}{n}$ converges for all real values of x except integral multiple of 2π.

Group-C

8. (a) Prove that any two bases of a finite dimensional vector space have the same number of elements.
   (b) Prove that the set (1, i, 0), (2i, 1, 1), (0, 1 + i, 1 - i) is a basis for $V_3(C)$.
9. (a) If W_1 and W_2 are two subspaces of a finite dimensional vector space V over a field F then show that $\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$.
10. (a) Find the basis and dimension of the subspace W of R^4 generated by the Vector (1, -4, -2, 1), (1, -3, -1, 2) and (3, -8, -2, 7)
   (b) Show that $T : V_2 (R) \rightarrow V_3 (R)$ defind by T(a, b) = (a + b, a – b, b) is a linear transformation.

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Nalanda Open University
Annual Examination - 2016
B.Sc. Mathematics (Honours), Part-II
Paper-IV

Time: 3.00 Hrs. **Full Marks: 80**

Answer any *five* Questions, selecting at least one question from each group.

Group-A

1. (a) Solve any two of the following differential equations

(i) $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\frac{dy}{dx} = y^2$ (ii) $y = 2px + p^2$

(iii) $(px - y)(py + x) = h^2 p$

(b) Find the orthogonal trajectory of $x^{2/3} + y^{2/3} = a^{2/3}$

2. (a) Solve $\frac{d^2y}{dx^2} + a^2 y = Sec\, ax$

(b) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2 e^{2x} Sin 2x$

3. (a) Solve $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x+2)y = x^2 e^x$

(b) Solve by using the method of variation of parameters $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3 e^x$

Group-B

4. (a) Prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\,\vec{b}\,\vec{c}]$

(b) Prove that $[\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

5. (a) If $\vec{a}$ *is a* unit vector, then prove that $\left|\vec{a}\times\frac{d\vec{a}}{dt}\right| = \left|\frac{d\vec{a}}{dt}\right|$

(b) If $\vec{r_1} = t^3\vec{i} + t^2\vec{j} + t\vec{k}$ *and* $\vec{r_2} = (t+1)\vec{i} + (t+2)\vec{j} - 3t\vec{k}$

Then find $\frac{d}{dt}(\vec{r_1}.\vec{r_2})$ *and* $\frac{d}{dt}(\vec{r_1}\, x\, \vec{r_2})$ *when* $t = 2$

6. (a) Prove the curl $(\vec{a}\, x\, \vec{b}) = \nabla\, x\, (\vec{a}\, x\, \vec{b}) = (\vec{b}.\nabla)\vec{a} - (\vec{a}.\nabla)\vec{b} + \vec{a}\, div\, \vec{b} - \vec{b}\, div\, \vec{a}$

(b) If $\phi = \log(x^2 + y^2 + z^2)$ *then find div(grad* ϕ*) and Curl (grad* ϕ*)*

Group-C

7. State and prove Gree's theorem.
8. (a) Find the equation of line of action of a system of co-planar forces.
(b) The forces P,Q,R act along the sides of a triangle formed by lines x+y=1, y-x=1 and y=2. Then find the equation of the line of action of the resultant.

Group-D

9. State and prove the necessary and sufficient condition for the principle of virtual work.
10. (a) State Hook's law and prove that work done against the tension in stretching a light elastic string is equal to the product of its extension and the means of the initial and final tensions.
(b) An elastic string without weight of which the unstretched length is *l* and the modulus of elasticity is the weight of n ozs is suspended by one end a mass of m ozs is attached to the other. Show that the vertical oscillation is $2\pi\sqrt{\frac{ml}{ng}}$.

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Nalanda Open University
Annual Examination - 2016
B.Sc. Mathematics (Subsidiary), Part-II
Paper-II

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Eight questions, selecting atleast one from each group. All questions carry equal marks.

Group-A

1. (a) Evaluate any two of the following integrals :

(a) $\int \frac{dx}{(x^2+1)\sqrt{1-x^2}}$ (b) $\int \frac{x^2 dx}{3\sqrt{x+2}}$ (c) $\int \frac{dx}{\sqrt{(1+x)}+\sqrt{x}}$

2. (a) Evaluate the integral as limit of sum (a) $\int_a^b \cos x dx$ (b) $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\, dx$

3. (a) Find the reduction formula for $\int (\sin^n x) dx$

(b) Evaluate $\lim_{n\to\infty} \sum \frac{1}{n}\sqrt{\frac{n+r}{n-r}}$

4. (a) Find the whole length of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

(b) Find the area between the curve $y^2(2a-x) = x^3$

5. Find the volume of the solid generated by revolving one arc of the cycloid *x = a(□ + Sin□), y = a(1 + Cos□) about its base*

6. Solve any two of the following differential equations :

(a) $y = 2px + p^2 y$ *(b)* $y = px + \frac{a}{p}$ *(c)* $p^2 - 7p + 12 = 0$

7. Find the general solution of the any two of the following equations :

(i) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$ (ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = xe^{3x}$

Group – B

8. Prove that the equation $x^2 + y^2 + z^2 - 4x + 2y - 6z - 22 = 0$ represent a sphere. Find the centre and the radius of the sphere.

9. (a) Find the equation of the right circular cylinder which passes through the circle

$x^2 + y^2 + z^2 = 9, x - y + z = 3.$

(b) Find the equation of through circular cylender whole axis is $\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$ and which passes through the point (0, 0, 3)

Group – C

10. (a) Define a convex set $S \subseteq R^n$ and prove that the sphere is a convex set.

(b) Prove that a hyper plane is a closed set.

11. Write short notes on the following :-

(a) Interior point (b) Closed set (c) Hyper plane

[See Back Page]

Group – D

12. Three forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation to the line of action of the resultant.

13. Find the equation of line of action of the resultant of a system of coplanar forces.

Group – E

14. Two uniform rods AB and AC, smoothly joined at A arc in equilibrium in vertical plane B and C rest on a smooth horizontal plane and middle points of AB and AC are connected by a string. Show that the tension of the string is $\frac{w}{\tan B + \tan C}$ where w is the weight of each rod.

15. (a) Define simple Harmonic motion (S.H.M). Derive the displacement in terms of time consumed for S.H.M.

    (b) Find the radial and transverse velocities and accelerations for a particle moving along a plane curve.

16. (a) A particle moving in a straight line is acted on by a force which works at a constant rate and charges its velocity from u to v in passing over a distance 'a'. Prove that time taken is $\frac{3(u+v)\ a}{2(u^2 + uv + v^2}$

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Nalanda Open University

Annual Examination - 2016

B.Sc. Mathematics (Honours), Part-III

Paper-V

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) State and prove Holder's inequality.
   (b) Let (X, *d*) be a metric space then show that the intersection of a finite number of open sets in x is opens.
2. (a) Let (X, *d*) be a metric space of E $\subseteq$ X, then show that E is closed if and only if E = $\overline{E}$
   (b) If A and B are subsets of a metric space (X, *d*) then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$
3. (a) Prove that a subset Y of a metric space (X, *d*) is closed iff the limit of any convergent sequence of points in Y is in Y.
   (b) If A and B are subset of (X, *d*) then show that
   (i) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$ (ii) $A^\circ \cap B^\circ \subseteq (A \cap B)^\circ$
4. (a) Prove that any convergent sequence in a metric space (x, d) is cauchy sequence.
   (b) Prove that if (X,d) be a complete metric space then a complete subspace of X is a closed subset of X
5. (a) Define the subsequence of a given sequence. Prove that in a metric space every subsequence of a convergent sequence is convergent and has the same limit.
   (b) Prove that (R^n,d) is a complete metic space.

Group 'B'

6. Let X be a non-empty set and {Ti} i$\in$I be any family of topologies on X then show that $\bigcap_{i \in I} T_i$ is also a topology on X.
7. Prove that if Y is a sub space of X and Z is a sub space of Y then Z is a sub space of X.

Group 'C'

8. State and prove a necessary and sufficient condition for the R-integrability of a bounded function F over an interval I=[a,b].
9. Prove that is a function f is continuous on [a,b] then it is R-integrable on [a,b].

Group 'D'

10. Using Cauchy's Integral test show that the series $\sum_{n=2}^{\alpha} \frac{1}{n(\log n)^p}$ converges if p>1 and diverges if 0<p≤1.
11. Define a normed linear space. If N is a non-zero normal linear space then show that N is a Banach space iff {x: ||x||=1} is complete.

Nalanda Open University

Annual Examination - 2016

B.Sc. Mathematics (Honours), Part-III

Paper-VI

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Prove that centre of a group G is a normal subgroup of G
   (b) If f is a function on a group G defined by $f(x)=x^{-1}$ for each x in G then show that f is an automorphism if and only if G is abelian.
2. (a) If A be a subgroup of a group G then prove that Ng(A) is a subgroup of G and A is a normal subgropu of Ng(A).
   (b) If C be the centre of a group G and T is any automorphism of G then prove that $T(C) \subseteq C$
3. (a) If f:R–T be a homomorphism of a ring R onto ring T then f is isomorphism iff kerf={0}.
   (b) Prove that the intersection of two ideals on a ring R is also an ideal on R.

Group 'B'

4. (a) If R is a commutative ring with unity element then show that R is a field if and only if it has non-trivial ideals.
   (b) If I is an ideal in a ring R then R(I) is a ring. Further if R is commutative then R/I is also commutative. Prove it.
5. (a) Let F be a field and f(x), g(x) are elements of F[x]. Then show that there exist polynomials s(x) and t(x) in F[x] such that
   $$g.c.d(f(x),g(x))=f(x)s(x)+g(x)t(x)$$
   (b) Let $f(x)=x^4+x^3-3x^2-x+2$ and
   $g(x)=x^4+x^3-x^2+x-2$
   Then find the g.c.d. of f(x) and g(x) as polynomials over Q

Group 'C'

6. State and prove Schroder-Bernstein theorem.
7. (a) Prove that $2^{\lambda^\circ} = C$

Group 'D'

8. (a) State and prove Zorn's Lemma.
   (b) Determine the order type of Q ∩ (-1,1), taken within natural order.
9. (a) Prove that ${}^n p_r = n \times {}^{n-1}p_{r-1}$
   (b) Find the solution of the recurrence relation
   $a_n=a_{n-1}+2a_{n-2}$ with initial condition
   $a_0=2$ and $a_1=7$
10. (a) Prove that real and imaginary parts of an analytic functions satisfy Laplace's equation.
    (b) If f(z) is an analytic functions z then show that
    $$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$$

Nalanda Open University
Annual Examination - 2016
B.Sc. Mathematics (Honours), Part-III
Paper-VII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any five questions, selecting at least one question from each group. All questions carry equal marks.

Group 'A'

1. (a) Test for the convexity of the Set
$S=\{(x_1, x_2) : x_1, x_2 \le 1; x_1 \ge 0, x_2 \ge 0\}$
(b) Solve the following LPP problem by graphical method.
Maximize $z = 2x + 5y$
Subject to the constraints
$x + 4y \le 24$, $3x + y \le 21$, $x + y \le 9$, where $x, y \ge 0$
2. (a) Prove that the set of all feasible solutions of a linear programming problem constitute a convex set.
(b) State and prove minimax Theorem.
3. Use simplex method to solve the following linear programming problem :
Maximize $z = 5x_1 + 7x_2$
Subject to the condition $2x_1 + 3x_2 \le 13$, $3x_1 + 2x_2 \le 12$ and $x_1 \ge 0$, $x_2 \ge 0$.

Group 'B'

4. (a) Solve $(y + z)\, dx + (z + x)dy + (x + y)dz = 0$.
(b) Solve $\frac{dx}{y^2 + yz + z^2} = \frac{dy}{z^2 + zx + x^2} = \frac{dz}{x^2 + xy + y^2}$.
5. Solve $(x^2y - y^3 - y^2x)dx + (xy^2 - x^2z - x^3)dy + (xy^2 + x^2y)dz = 0$.
6. Solve $\frac{dx}{dt} + 5x + y = e^t$, $\frac{dy}{dt} - x + 3y = e^{2t}$
7. Using Charpit's method, solve $px + qy = pq$

Group 'C'

8. (a) Find the attraction of a circular disc at an external point at height h.
(b) Prove that the attraction of a solid hemisphere at the centre of its plane base is $\frac{3}{2} \nu \frac{M}{a^2}$
9. (a) Find the potential of a circular disc at a point distant h on the axis from the centre.
(b) Prove that the potential of a uniform circular cylinder of length 2a radius of the base a and mass M at the middle point of its axis is :
$$\frac{\nu M}{a}\left[(\sqrt{2} - 1) + \log(\sqrt{2} + 1)\right]$$

Group 'D'

10. (a) Find the pressure at a point at a depth h below in a heavy homogeneous fluid from the free surface.
If three fluids whose diversities are in A.P fill a semi circular tube whose bounding diameter is horizontal, show that the depth of one of the common surfaces is double that of the other.
(b) A hollow weightless hemisphere filled with liquid is suspended freely from a point in the rim of its base. Show that the thrust on the base is to the weight of the contained liquid as $12 : \sqrt{73}$
11. Find the centre of pressure of a vertical circle of radius a wholly immersed in homogeneous liquid with its centre at a depth h below the free surface.

Nalanda Open University

Annual Examination - 2016

B.Sc. Mathematics (Honours), Part-III

Paper-VIII

Time: 3.00 Hrs. **Full Marks: 80**

Answer any Five questions. All questions carry equal marks.

1. (a) Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^4}{4}}$

   (b) Evaluate : $\left(\frac{\Delta^2}{E}\right)x^3$

2. (a) Prove that the divided difference are symmetrical in all their arguments.

   (b) Prove that

   $$u_0 + u_1 x + u_2 x^2 + ... = \frac{u_0}{1-x} + \frac{x\Delta u_0}{(1-x)^2} + \frac{x^2\Delta^2 u_0}{(1-x)^3} + ...$$

3. Obtain the estimates the missing terms in the following table :

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
y(x)	0.135	–	0.111	0.100	–	0.082	0.074

4. (a) Find the sum of :

   $$\frac{1}{(51)^2} + \frac{1}{(53)^2} + \frac{1}{(55)^2} + ... + \frac{1}{(99)^2}$$

   (b) Drive the general quadrature formula for equidistant value of x.

5. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using simpson's $\frac{1}{3}rd$ and $\frac{3}{8}th$ rule. Find the approximate value of π

6. Solve the equations.

   (a) $16u_{x+2} - 8u_{x+1} + u_x = 0$

   (b) $u_{x+2} - 4u_{x+1} + 13u_x = 0$

7. Solve the equations.

   (a) $u_{x+2} - 3u_{x+1} + 2u_x = (2)^x$

   (b) $u_{x+1} = (2)^x u_x$

8. (a) Describe the Picard's method of successive approximation for the solution of the differential equation $\frac{dy}{dx} = f(x,y)$

   (b) Solve the differential Equation $\frac{dy}{dx} = x + y$ with initial condition y(0) = 1 by Runge-Kutta rule from x = 0 to x = 0.4 and h = 0.1

9. (a) Use Bisection Method to find solution of the Equation $x^3 - 9x + 1 = 0$ between x = 2 and x = 4.

   (b) Give geometrical description of Newton-Raphsm's Method.

10. (a) Explain Relaxation method for finding the solution of a system of linear algebraic equation AX=B.

    (b) Find the real root of the Equation $x \log_{10} x - 1.2 = 0$ with the help of Reguli falsi Method.