NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–I

(Advanced Abstract Algebra)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Jordan-Holder theorem on any group.

2. Define Homomorphism and Kernel of homomorphism from a module M into a module N. If f is a module homomorphism then f is an isomorphism if and only if $K(f) = 0$. Prove this.

3. Establish the transitivity property of finite extension of a field.

4. State and prove Kronecker's theorem.

5. State and prove fundamental theorem of Galois theory.

6. (a) Define algebraic and simple extension of a field and give an example of each one.
   (b) If a and b are algebraic over a field F then prove that $a \pm b$, ab, $ab^{-1}\,(b \neq 0)$ are also algebraic over F.

7. (a) Define a subnormal series of a group. Hence or otherwise form a subnormal series of the additive group of integers.
   (b) Construct all composition series of Z_{60}.

8. State and prove Zassenhauss Butterfly Lemma.

9. (a) Define a submodule of a module M. Show that arbitrary intersection of submodules of a module M is a submodule of M.
   (b) Show that a module M is the direct sum of two submodules M_1 and M_2 iff (i) $M_1 + M_2$ and (ii) $M_1 \cap M_2 = \{0\}$.

10. (a) Find the Galois group of the equation $x^3 - 2 = 0$ over the field Q of rational numbers.
    (b) Prove that if $K = \phi(\sqrt{2})$ where ϕ is the field of all rational numbers then ϕ is the fixed field under the group of automorphism of K.

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Examination Programme, 2017

M.Sc. Mathematics, Part-I

Date	***Papers***	***Time***	***Examination Centre***
12.05.2017	Paper-I	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
16.05.2017	Paper-II	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
18.05.2017	Paper-III	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
20.05.2017	Paper-IV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
22.05.2017	Paper-V	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
24.05.2017	Paper-VI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
26.05.2017	Paper-VII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
29.05.2017	Paper-VIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–II

(Real Analysis)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove a necessary and sufficient condition for a function f to be R-integrable over [a, b].

   (b) If $f_1, f_2 \in R(\alpha)$ on [a, b] then prove that $f_1 + f_2 \in R(\alpha)$ and $c\,(f_1 + f_2) \in R(\alpha)$ for every constant c and $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$ *and* $\int_a^b cf d\alpha = c\int_a^b f d\alpha$.

2. (a) If $f \in R\ (\alpha)$ and α is monotonically increasing on [a, b], then show that $|\,f\,| \in R(\alpha)$ on *[a, b]* and $\left|\int_a^b f\, d\alpha\right| \le \int_a^b |\,f\,|\, d\alpha$.

   (b) If f is an arbitrary function and α be a constant function on [a, b]. Prove that $f \in R(\alpha)$ on [a, b] and $\int_a^b f\, d\alpha = 0$.

3. Show that a function f defined on [a, b] is of bounded variation iff it can be represented as difference of two monotonically increasing functions on [a, b].

4. (a) State and prove Cantor's intersection theorem.

   (b) Show by an example that Heine Borel theorem can not be extended to unbounded intervals.

5. Let $f : R^2 \to R$ be defined by $f(x, y) = \dfrac{xy\,(x^2 - y^2)}{x^2 + y^2}$ if $(x, y) \ne (0, 0)$ and $f(0, 0) = 0$, then show that $D_{1,2} f(0, 0) \ne D_{2,1}\ f(0, 0)$.

6. State and prove Abel's theorem.

7. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \dfrac{\lfloor n}{n^n} x^n$.

8. What do you mean by the Extreme values of a function and find these values for the function defined by $f(x, y, z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$.

9. State and prove inverse function theorem.

10. Find $\dfrac{\partial(y_1, y_2, y_3, ---, y_n)}{\partial(x_1, x_2, x_3, ---, x_n)}$ where $y_1 = x_1\,(1 - x_2)$; $y_2 = x_1 x_2\,(1 - x_3)$; $y_3 = x_1\, x_2\, x_3\,(1 - x_4) ---$; $y_{n-1} = x_1 x_2 x_3\ ---\ x_{n-1}(1 - x_n)$, $y_n = x_1 x_2 x_3\ ---\ x_n$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)
Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If E_1 and E_2 are measurable, sets, then show that $E_1 \cup E_2$ *and* $E_1 \cap E_2$ are measurable sets.
   (b) Prove that a Denumerable set is measurable with measure zero.
2. (a) If (E_n) is a sequence of measurable sets such that $E_1 \supseteq E_2 \supseteq E_3 \supseteq$ .... and $E = \bigcap_{i=1}^{\infty} E_i$ and $m(E_1) < \infty$ then show that $m(E) = m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{i \to \infty} it\ m(E_i)$.
   (b) Show that the Lebesgue measure of the set $\{ x \in R : 0 < x < 1$ and x has a decimal expansion not using the digit 7}, is zero.
3. (a) Prove that if f is a measurable function then $|f|$ is also a measurable function.
   (b) If f and g are measurable functions then prove that $f \pm g$, fg are also measurbale.
4. Prove that the class of measurbale functions is closed with respect to all analytic operations.
5. Define the Lebesgue integral of a function in details. If *f, g* are L-integrable then show that $f + g$ in L-integrable and $\int (f + g) = \int f + \int g$.
6. (a) If f is L-integrable, then prove that $|f|$ is also L-integrable and show that $\left|\int f\right| \leq \int |f|$.
   (b) Examine the L-integrability of $f(x) = \frac{d}{dx}\left(x^2 \sin\frac{1}{x^2}\right)$ over [0, 1].
7. State and prove Lebesgue Monotone convergence theorem.
8. State and prove Fatou's Lemma.
9. State and prove Dominated convergence theorem.
10. (a) Show that a function of bounded variation over [a, b] is bounded.
    (b) Show that an absolutely continuous function is continuous but the converse is not necessarily true.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that a topological space is T_0-Space, *iff* $x, y \in X$ and $x \neq y \Rightarrow \{x\} \neq \{y\}$.
   (b) Show that the property of a T_1-space is both hereditary and topological.

2. (a) Prove that a topological space is normal *iff* each neghourhood of a closed set *F* contains the closure of some neighbourhoods of *F*.
   (b) Show that every metric space is a normal space.

3. (a) Show that every compact sub-space of the real line is closed and bounded.
   (b) Prove that a compact Hausdorff space is regular.

4. If X and Y are topological spaces, then show that X × Y is connected *iff* X and Y are connected.

5. (a) Define a T_3-space and T_4-space. Prove that every T_4-space is a T_3-space.
   (b) Prove that every compact sub-space of a Hausdorff space is closed.

6. (a) Give an example of Topological space which is a T_1-space but not a T_2-space.
   (b) Show that a finite sub-set of T_1-space has no cluster point.

7. If $\{T_i\}$ is a family of Topologies on a non-empty set *X*, then prove that $\bigcap_{i \in I} T_i$ is also a topology on *X*.

8. Prove that a necessary and sufficient condition for a one to one mapping $f : X \rightarrow Y$ to be a homeomorphism is that $f(\overline{A}) = \overline{f(A)}$ for every $A \subseteq X$, where *X* and *Y* are Topological spaces.

9. (a) Prove that in a Hausdorff space every convergent sequence has a unique limit.
   (b) If *A* is a sub-set of a Topological space *(X, T)* then show that
   (i) $(Int\ A)' = \overline{A}'$, (ii) $(\overline{A})' = Int(A')$

10. (a) Introduce the concept of connectedness and disconnectedness of topological spaces. Prove that a topological space is connected *iff* ϕ and X are its only subsets which are both open and closed.
    (b) Show that the open interval (0, 1) on the real line *R* is not compact.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that a partially order set $(P(X), \subseteq)$ is a lattice.
   (b) Let R be a ring and L be the lattice of all ideals of R, then prove that L is a modular.

2. (a) Define isomorphism between two lattices.
   (b) Prove that a necessary and sufficient condition for a one-one onto mapping f between two lattices to be isomorphism, is that f and f^{-1} are both order preserving.

3. Prove that a close-open set G having $0 = \phi$ and $1 = x$ is a Boolean Algebra.

4. (a) Show that the relation precedes $(x \leq y)$ in a Boolean algebra B is a partial order relation.
   (b) If B is a Boolean algebra then prove that $\forall\, x,\ y \in B$ the following are equivalent
   (i) $x \wedge y' = 0$ (ii) $x \vee y = y$ (iii) $x' \vee y = 1$ (iv) $x \wedge y = x$

5. (a) Prove that in Boolean Algebra, the complement of an element is unique.
   (b) Prove that a Boolean Algebra B is a complement distributive lattice.

6. Prove that a linear operator E is a projection on some sub-space *iff* it is an idempotent.

7. Let $V(F)$ be a finite dimensional vector space and W is a sub-space of V, then show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

8. Prove that two dual quadratic forms are equivalent *iff* they have the some rank and index.

9. (a) Show that a Hermitian form remains Hermitian by a non-singular transformation.
   (b) Let {(1, 1, 1), (0, 1, 1), (0, 0, 1)} be a basis of Euclidean space R^3, find its orthonormal basis.

10. (a) Convert $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.
    (b) Show that the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of index 3.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–VI

(Complex Analysis)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Cauchy's theorem and describe some application in one or two cases.

2. State and prove Poisson's integral formula.

3. Using Cauchy integral formula evaluate $\int \frac{z\,dz}{(9 - z^2)(z + 1)}$.

4. Find Taylor's expansion of the functions $f(z) = \frac{z}{z^2 + 9}$ around $z = 0$ also determine the radius of convergence of Taylor's series so developed.

5. (a) Derive the necessary and sufficient condition for analyticity of $f(z)$.

   (b) Show that $u = ½\log(x^2 + y^2)$ is a harmonic function. Find the analytic function $f(z)$ whose real part is u.

6. (a) What is a power series ? State and prove Cauchy-Hadmard theorem.

   (b) Determine the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{2^n + 1}$.

7. (a) Describe different kinds of singularity.

   (b) What is pole of a function also introduce the residue at simple pole and pole of order m.

8. Evaluate the following integrals :—

   (a) $\int_0^{2\pi} \frac{d\theta}{1 + a\,Cos\theta}$, *where* $a^2 < 1$

   (b) $\int_0^{\infty} \frac{dx}{(1 + x^2)^2}$

9. (a) State and prove the necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.

   (b) Show that the transformation $w \; \frac{5 - 4z}{4z - 2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w –plane and hence find its centre.

10. By introducing Bilinear Transformation, derive the existence of fixed points of a Bilinear Transformation.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Compute the first three successive approximations for the solution of the equation $y' = y^2,\ y(0) = 1$.

   (b) Find an interval I containing Γ and a solution g of $y' = \frac{dy}{dx} = f(x,\ y)$ on I satisfying $g(\Gamma) = s$.

2. State and prove Picard's-Lindelof theorem.

3. (a) Determine the constants M *and* C and x for the initial value problem $y' = y,\ y(0) = 1$, $R = \{(x,\ y) : |\,x\,| \le 1 \text{ and } |\,y - 1\,| \le 1\}$.

   (b) Find e^A if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.

4. (a) Explain different types of critical points for a system and give the geometrical meaning of each critical point.

   (b) Find the nature of critical point (0, 0) of the system $\frac{dx}{dt} = x + 5y,\ \frac{dy}{dt} = 3x + y$ and discuss their stability.

5. (a) Explain the nature of critical point of a non-linear system $\frac{dx}{dt} = ax + by + \phi(x,\ y)$ and $\frac{dy}{dt} = cx + dy + \psi\ (x,\ y)$.

   (b) Determine the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = Sin x - 4y;\ \ \frac{dy}{dt} = Sin 2x - 5y$.

6. Define Lipschitz condition in a region. Show that the following function does not satisfy the Lipschitz condition in the region indicated : $f(x,\ y) = \frac{\sin y}{x},\ \ f(0,\ y) = 0,\ \ |\,x\,| \le 1,\ \ |\,y\,| \le \infty$.

7. Prove that a necessary and sufficient condition that a solution matrix G be a fundamental matrix, is that $G(x) \ne 0$ for $x \in I$.

8. Compute Rodrigue's formula for Legendre polynomial.

9. Derive the expression for the generating function for Bessel's function.

10. Prove that,

    (a) $J_{-n}(x) = (-1)^n\ J_n(x)$, where n is a +ve integer.

    (b) $J_n(-x) = (-1)^n\ J_n(x)$, where n is a positive or a negative integer.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define isomorphism between two graphs and give two examples of isomorphic graphs.
   (b) Determine the difference between a circuit and Eulerian circuit.
2. (a) Prove that a pseudograph is Eulerian iff it is connected and every vertex is even.
   (b) Prove that a complete graph of n vertices is planar if $n \leq 4$.
3. (a) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.
   (b) If g is a connected graph with e-edges and v-vertices, then prove that $e \leq 3v - 6$.
4. (a) Prove that $2^{N_0} = C$, where N_0 is the cardinal number of the set N, where as C is the cardinal number of [0, 1].
   (b) State and prove Schroder-Bernstein theorem.
5. (a) For any three cardinal number α, β, γ; show that (i) $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta + \gamma}$, (ii) $(\alpha\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$.
   (b) What is Axiom of choice ? Show that the Axiom of choice is equivalent to Zermelo's postulates.
6. (a) If A and B are two countable sets, then show that A × B is also countable.
   (b) Define countable set. Prove that the interval [0, 1] is uncountable.
7. (a) State and prove the division algorithm of integers.
   (b) Define congruency between two integers under a positive integer m. Prove that the relation $a \equiv b$ (mod m) defines an equivalence relation on the set of integers.
8. (a) State and prove Chinese remainder theorem.
   (b) Show that $(a, m_1) = 1, (a, m_2) = 1 \Leftrightarrow (a, m_1 m_2) = 1$.
9. State and prove Fermat's theorem.
10. (a) What is a circular helix ? Find the osculating plane at the point $P(\theta)$ on the helix $x = a\,Cos\theta,\ y = a\,Sin\,\theta,\ z = c\theta$.
    (b) Prove that $\left[\vec{r}^{\,I}, \vec{r}^{\,II}, \vec{r}^{\,III}\right] = \dfrac{T}{\rho^2}$, where $\vec{r}$ the current point, T is torsion and ρ is the radius of curvature.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Describe Newton's iterative formula to find a square root and a inverse square root of a number.
   (b) Find all the real roots of the equation $x^2 + 4 Sin\, x = 0$ correct to four places of decimals by using Newton-Raphson method.

2. (a) Find a positive root of $xe^x - 1 = 0$ lying between 0 and 1, using iteration method.
   (b) Find *f(6)*, when *f(0) = 3, f(1) = 6, f(2) = 8, f(3) = 12* and the third difference being constant.

3. Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.03$ by Newton's backward difference formula using the following table :—

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

4. Determine the value of the integral $\int_4^{5.2} \log x \, dx$ by Trapezoidal rule.

5. Find the formula for Quadrature for equally spaced arguments and hence derive Simpson's three-eighth rule.

6. Solve the equation,
   $y_{x+3} + y_{x+2} - y_{x+1} - y_x = 0$, where $y_0 = 2,\ y_1 = -1,\ y_2 = 3$.

7. Form the difference equation corresponding to the family of curves $y_x = ax^2 + bx - 3$.

8. Find the polynomial of lowest degree which is interpolated by the following sequence of numbers using any method : 0, 7, 26, 63, 124, 215, 342, 511.

9. Define factorial notation and prove that $(x)^{(-n)} = \frac{1}{(x + hn)^{(n)}}$ where *h* is the interval of differencing.

10. (a) Prove that $(1 + \Delta)(1 - \nabla) = 1$.
    (b) Compare Newton's method with Regula-Falsi method. Apply Newton's Raphson method to find square root of 12 to five places of decimal.

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Examination Programme, 2017

M.Sc. Mathematics, Part-II

Date	***Paper***	***Time***	***Examination Centre***
01.06.2017	Paper–IX	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
03.06.2017	Paper–X	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
05.06.2017	Paper–XI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
07.06.2017	Paper–XII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
09.06.2017	Paper–XIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
13.06.2017	Paper–XIV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
15.06.2017	Paper–XV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
17.06.2017	Paper–XVI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then prove that dual of l_p is l_q.

2. State and prove Hahn Banach theorem.

3. Sate and prove F. Riesz's lemma.

4. Define a normed linear space and Banach space. Also prove that the following inequality in case of normed linear space holds $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

5. (a) Show that, $x_n \to x$ $w.r.t.$ $\| \quad \|$ if and only if $x_n \to x'$ $w.r.t.$ $\| \quad \|'$.
   (b) If X and Y be two normed linear spaces and X be finite dimensional, then show that every linear map from X to Y is continuous.

6. Define a quotient space. Let M be a closed linear sub space of a normed linear space N. If the norm of a coset $x + M$ in the quotient space $\frac{N}{M}$ is defined by $\| x + M \| = \inf \{ \| x + M \| : x \in M \}$. Then show that $\frac{N}{M}$ is a normed linear space. If N is a Banach space, then show that $\frac{N}{M}$ is also a Banach space.

7. (a) If M and N are closed linear sub spaces of a Hilbert space H such that $M \perp N$ then prove that the linear sub space M + N is also closed.
   (b) Let L be a linear space over F. Then show that the sum of two inner products on L is an inner product on L.

8. State polarization identity and explain about it in an inner product space.

9. Give an example of a Banach space which is not a Hilbert space.

10. (a) If X and Y are Banach spaces and T is a continuous linear transformation of X into Y then show that T is an open mapping.
    (b) If the mapping $T \to T'$ is a norm preserving mapping of $\beta(N)$ to $\beta(N^*)$, then prove that,
    (i) $(\alpha T_1 + \beta T_2)^* = \alpha T_1^* + \beta T_2^*$, and
    (ii) $(T_1 T_2)^* = T_2^* T_1^*$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)
Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Use the method of separation of variables to solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

2. (a) Solve the boundary value problem $\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial y}$, when $u(0, y) = 8e^{-3y}$.

   (b) Show that the family of surfaces defined by $x^2 + y^2 =$ constant is a family of equipotential surfaces in free space and then, find the law of potential.

3. (a) Describe the Jacobi's method for the solution of the partial differential equation $F(x, y, z, p, q) = 0$.

   (b) Solve the partial differential equation $xyr + x^2 s - yp = x^3 e^y$.

4. Reduce the partial differential equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its solution.

5. Explain Charpit's method for the solution of non-linear partial differential equations of the first order.

6. Solve the following partial differential equations using Charpit's method.

   (i) $(p^2 + q^2)y = qz$ (ii) $2zx - px^2 - 2qxy + pq = 0$

7. Solve,

   (i) $(D - D'^2)z = Cos\ (x - 3y$ (ii) $(D^2 - DD' + D' - 1)z = Cos(x + 2y) + e^y$.

8. Find the general solution of the partial differential equation
   $px(x + y) - qy(x + y) = (x - y)(2x + 2y + z)$.

9. (a) Derive the Fourier equation of heat conduction.

   (b) A rod of length ℓ with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to *0°c* and are kept at that temperature. Find the temperature *u (x, t).*

10. Solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le \ell;\ t \ge 0$. Subject to the boundary conditions $\left.\begin{array}{l} u(0, t) = 0,\ t > 0 \\ \frac{\partial u}{\partial x}(\ell, t) = 0,\ t > 0 \end{array}\right\}$ and initial conditions $u(x, 0) = \begin{cases} x, & 0 \le x < \frac{\ell}{4} \\ \frac{\ell}{2} - x, & \frac{\ell}{4} \le x < \frac{\ell}{2} \\ 0, & \frac{\ell}{2} \le x < \ell \end{cases}$ and $\frac{\partial u}{\partial t}(x, 0) = 0,\ 0 \le x < \ell$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XII

(Analytical Dynamics)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What is Hamilton's function ? Find the differential equations for Hamilton's function.

2. Discuss the motion of spherical pendulum deducing from Hamilton's canonical equations of motion.

3. Derive Lagrange's equation of motion from Hamilton's canonical form of equations.

4. Derive the formula for kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.

5. (a) Show that Lagrange Bracket does not obey the commutative law of algebra.

   (b) Prove that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical. Find the generating function *F(q, Q)*.

6. (a) Derive Hamilton-Jacobi equation and then find Hamilton's characteristic function.

   (b) Give the physical significance of Hamilton characteristic function.

7. Discuss the motion of a sphere when the small sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius.

8. Find the equation of motion of a simple pendulum applying Lagrange's equation of motion.

9. State and prove Jacobi-Poisson theorem.

10. (a) Describe the motion of particle about revolving axes.

    (b) Using invariance of Bilinear form show that the transformation $Q = \frac{1}{p}$ and $P = p^2 q$ is canonical.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Derive the equation of continuity in Cartesian form.

2. Derive the equation of motion under impulsive force.

3. Derive Euler's equation of motion in cylindrical polar co-ordinates.

4. Describe the motion of a fluid between rotating co-axial circular cylinders.

5. (a) Derive the rate of strain tensor of fluid in motion.
   (b) Show that the velocity field defined at a point P by $(1+2y-3z,\ 4-2x+5z,\ 6+3x-5y)$ represents a rigid body rotation.

6. (a) A velocity field is given by $\vec{q} = -x\,\hat{i} + (y+t)\,\hat{j}$. Find the stream function and the stream lines for field at $t = 2$.
   (b) Describe Source, Sink and Doublet and give their suitable examples.

7. Prove that the liquid motion is possible when velocity at (x, y, z) is given by $u = \dfrac{3x^2 - r^2}{r^5}$, $v = \dfrac{3xy}{r^5}$, $w = \dfrac{3xz}{r^5}$.

8. A velocity field is given by $\vec{q} = \dfrac{x\,\vec{j} - y\,\vec{i}}{x^2 + y^2}$ calculate the circulation round the square having corners at (1, 0), (2, 0), (2, 1) and (1, 1). Also test for the flow of rotation.

9. (a) Write notes on the following :–
      (i) Velocity Potential
      (ii) Velocity Vector
      (iii) Boundary Surface
   (b) The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\,\hat{j} - z\,\hat{k}$. Determine the equations of streams passing through the point (1, 1, 1).

10. (a) Derive Navier-Stokes equation of motion of viscous fluid.
    (b) Use Navier's-Stokes theorem to find the equation of vorticity.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XIV

(Operation Research)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Define a convex set in R^n. Let S and T be two convex sets in R^n then for any scalars α, β show that $\alpha S + \beta T$ is also a convex set in R^n.

   (b) Prove that every Extreme point of the convex set of feasible solution is a B.F.S. (Basic Feasible Solution).

2. (a) If (2, 1, 3) is a F.S. of the set of equations $4x_1 + 2x_2 - 3x_3 = 1$, $6x_1 + 4x_2 - 5x_3 = 1$ then reduce the F.S. to B.F.S. of the set.

   (b) Solve the following L.P.P. problem by any method of your choice (except graphically)
   Max $z = 5x_1 + 7x_2$
   S.t. $x_1 + x_2 \le 4$, $3x_1 + 8x_2 \le 24$, $10x_1 + 7x_2 \le 35$ *and* x_1, $x_2 \ge 0$.

3. Solve the following L.P.P. problem by simplex method.
   Minimize $z = x_1 - 3x_2 + 2x_3$
   Subject to $3x_1 - x_2 + 2x_3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 8x_3 \le 10$; $x_1, x_2, x_3 \ge 0$.

4. Apply two phase simplex method to solve the following L.P.P.
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \ge 4$, $x_1 + 7x_2 \ge 7$; $x_1, x_2 \ge 0$.

5. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   S.t. $x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \le 3$, $2x_2 - x_3 \ge 4$; $x_1, x_2 \ge 0$; x_3 is unrestricted in sign.

6. (a) Prove that dual of the dual of a given primal is the primal itself.

   (b) If X_0 and W_0 are feasible solutions to the primal and dual respectively then prove that $c X_0 \le W_0 b$.

7. Solve the following L.P.P.
   Max $z = 10x_1 + 3x_2 + 6x_3 + 5x_4$
   S.t. $x_1 + 2x_2 + x_4 \le 6$, $3x_1 + 2x_3 \le 5$, $x_2 + 4x_3 + 5x_4 \le 3$ *and* $x_1, x_2, x_3, x_4 \ge 0$
   also compute the limits for a_{11} and a_{23} so that the new solution remains optimal feasible solution.

8. Solve the following assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	97
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

9. The pay-off matrix of a game is given below. Find the solution of the game for A and B.

		B				
		I	II	III	IV	V
	I	-2	0	0	5	3
A	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

10. Solve the following NLPP using the method of Lagrangian multipliers
    Min $z = x_1^2 + x_2^2 + x_3^2$
    Subject to constraints $4x_1 + x_2^2 + 2x_3 = 14$; $x_1, x_2, x_3 \ge 0$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV
(Tensor Algebra, Integral Transforms, Integral Equations, Operational Research Modeling)

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) A covariant tensor has components xy, $2y - z^2$, zx in rectangular co-ordinates. Determine its covariant components in spherical co-ordinates.
   (b) Express in the matrix notation the following transformation equations for (i) a covariant vector, (ii) a contra variant vector, (iii) a contra variant tensor of rank two assuming N = 3.
2. (a) Define symmetric and skew symmetric tensors. Prove that a symmetric tensor of rank two has at most $\frac{1}{2}N(N+1)$ different components in V_N. Where as a skew symmetric tensor of rank two has $\frac{N}{2}(N-1)$ independent components in V_N.
   (b) State and prove quotient theorem of tensors; give an example.
3. (a) Show that the covariant derivative of a co-variant vector is a mixed tensor of rank two.
   (b) Prove that the outer product of two tensors (r, s) and (p, q) types is a tensor of $(r + s)(p + q)$ type.
4. (a) If $L\{F(t)\} = f(s)$ then prove that
   $L\{F^n(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) \ldots\ldots\ldots\ldots - s F^{n-2}(0) - F^{n-1}(0)$.
   (b) Prove that Laplace transform of $\frac{Sin\, at}{t}$ is $Cot^{-1}\left(\frac{s}{a}\right)$.
5. (a) State and prove Convolution theorem on inverse Laplace transform.
   (b) Find the inverse Laplace transform of
   (i) $\frac{1}{s^2(s^2 + a^2)}$ (ii) $\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$
6. (a) Define Fredholm integral and voltera integral equations.
   (b) Prove that the function $u(t) + (1 + x^2)^{-1/2}$ is a solution of the voltera integral equation.
   $u(x) = \frac{1}{1 + x^2} - \int_0^x \frac{t}{1 + x^2} u(t)dt$.
7. Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x$ with the initial conditions $y(0) = 1$ and $y'(0) = -1$.
8. Describe deterministic model with instantaneous production, shortage allowed.
9. Solve $(D^3 - D^2 + 4D - 4)x = 68 e^t\, Sin\, 2t$.
10. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ and hence evaluate
    (i) $\int_{-\infty}^{\infty} \frac{Sin\, sa\; Cos\, sx}{s} ds$ (ii) $\int_0^{\infty} \frac{Sin\, s}{s} ds$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2017

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Discuss integer constant, floating point constant and character constant. What are the rules for constructing integer contents ?
2. What is control statement is C Language ? Explain with the help of an example.
3. What is the use of operator in C language ? Discuss Arithmetic operators, Relational operators and Assignment operators with example.
4. What is meant by looping in 'C' Language ? Discuss two different types of looping with example.
5. What is the purpose of the switch statement ? How does switch statement differ from the other statements ?
6. Describe different Format specifiers and Escape sequence along with their usage and examples.
7. What is function ? Are functions required when writing a C program ?
8. Describe two different ways to utilize the increment and decrement operators ? How do the two methods differ ?
9. Write a programme in C to find the roots of a quadratic equation.
10. Write short notes on any ***Two*** of the following :–
    (a) Two dimensional Array
    (b) Go to statement
    (c) Recursion
    (d) Local and Global Variables

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2017

Practical Counselling Programme

<table>
<tr><th>Enrollment No.</th><th>Date</th><th>Time</th><th>Venue</th></tr>
<tr><td>150290001 to 150290500</td><td rowspan="2">28.06.2017
to
06.07.2017</td><td>9.00 AM to 1.00 PM</td><td rowspan="2">School of Computer Education (IT)
Nalanda Open University, 12th Floor,
Biscomaun Tower, Patna-800001</td></tr>
<tr><td>150290501 to 150290838
& All Old Students</td><td>1.00 PM to 5.00 PM</td></tr>
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Practical Examination Programme

<table>
<tr><th>Enrollment No.</th><th>Date</th><th>Time</th><th>Venue</th></tr>
<tr><td>150290001 to 150290500</td><td>07.07.2017</td><td>10.30 AM to 1.30 PM</td><td rowspan="2">School of Computer Education (IT)
Nalanda Open University, 12th Floor,
Biscomaun Tower, Patna-800001</td></tr>
<tr><td>150290501 to 150290838
& All Old Students</td><td>07.07.2017</td><td>2.00 PM to 5.00 PM</td></tr>
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