NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–I
(Advanced Abstract Algebra)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that every finite extension of a field F is algebraic.
   (b) Prove that if F is a field and K is an extension of F then an element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.
2. (a) State and prove Remainder theorem.
   (b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
3. If Ψ be an isomorphism of a field F_1 onto a field F_2 such that $\alpha\Psi = \alpha'$ for every $\alpha \in F_1$ then prove that there is an isomorphism ϕ of $F_1[x]$ on to $F_2[t]$ with the property $\alpha\phi = \alpha\Psi = \alpha'$ for each $\alpha \in F_1$.
4. If F is a field of characteristics 0 and a, b are algebraic over F then prove that there exists an element $c \in F[a, b]$ such that $F[a, b] = F[c]$ i.e. $F[a, b]$ is a simple extension.
5. (a) Show that the splitting field of $x^4 + 1 = 0$ is $Q(\sqrt{2}, i)$ whose degree over Q is 4.
   (b) Find necessary and sufficient condition on a, b so that the splitting field of irreducible polynomial $x^3 + ax + b$ has degree 3 over Q.
6. Let F be a field of characteristics O. Then prove that a polynomial $f(x) \in F(x)$ is solvable by radicals over F if and only if its splitting field K over F has solvable Galois group $G[K, F]$.
7. (a) Show that every element in a finite field can be written as the sum of two squares.
   (b) Prove that the group $G[Q(\alpha), Q]$ where $\alpha^5 - 1$ and $\alpha \neq 1$, is isomorphic to cyclic group of order 4.
8. (a) Show that the Galois group of $x^4 + x^2 + 1$ is the same as that of $x^6 - 1$.
   (b) Let K be the splitting field of $x^n - a \in F[x]$. Then show that $G(K, F)$ is a solvable group.
9. (a) Prove that the necessary and sufficient condition for a module M to be a direct sum of its two sub modules M_1 and M_2 are that (i) $M = M_1 + M_2$, (ii) $M_1 \cap M_2 = \{0\}$.
   (b) State and prove Schur's theorem.
10. (a) Prove that every homomorphic image of a Noetherian (artinian) module is Noetherian (artinian).
    (b) Prove that every finitely generated module is homomorphic image of a finitely generated free module.

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Examination Programme, 2018

M.Sc. Mathematics, Part-I

Date	*Papers*	*Time*	*Examination Centre*
24.05.2018	Paper–I	3.30 PM to 6.30 PM	Nalanda Open University, Patna
26.05.2018	Paper–II	3.30 PM to 6.30 PM	Nalanda Open University, Patna
28.05.2018	Paper–III	3.30 PM to 6.30 PM	Nalanda Open University, Patna
30.05.2018	Paper–IV	3.30 PM to 6.30 PM	Nalanda Open University, Patna
01.06.2018	Paper–V	3.30 PM to 6.30 PM	Nalanda Open University, Patna
05.06.2018	Paper–VI	3.30 PM to 6.30 PM	Nalanda Open University, Patna
07.06.2018	Paper–VII	3.30 PM to 6.30 PM	Nalanda Open University, Patna
09.06.2018	Paper–VIII	3.30 PM to 6.30 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–II

(Real Analysis)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Let f be bounded and g a non-decreasing function on [a, b]. Then prove that $f \in RS(g)$ if and only if for every $\in > 0$ there exists a partition P of [a, b] such that $U(P, f, g) - L(P, f, g) < \in$

2. Let $f_1, f_2 \in RS(g)$ on [a, b] then prove that $f_1 + f_2 \in RS(g)$ on [a, b] and $\int_a^b (f_1 + f_2) dg = \int_a^b f_1 \, dg + \int_a^b f_2 \, dg$.

3. (a) Define a function of bounded variation clearly and prove that a bounded monotonic function is a function of bounded variation.

   (b) If $f \in BV[a, b]$ and $c \in [a, b]$ then prove that $f \in BV[a, c]$ and $f \in BV[c, b]$ and conversely moreover $V(f, a, b) = V(f, a, c) + V(f, c, b)$.

4. State and prove Implicit function theorem.

5. Discuss the continuity and differentiability of the function f defined by $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ at $(0, 0)$.

6. State and prove Schwarz's theorem for a function of two variables.

7. If f is defined by $f(x, y) = \dfrac{x^2 y^2}{x^2 + y^2}, \; x^2 + y^2 \neq (0, 0)$

   $= 0$ otherwise.

   Then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$ although neither f_{xy} nor f_{yx} is continuous at $(0, 0)$. Account for the equality.

8. Find the radius of convergence of the following power series.

   (i) $\sum_{n=1}^{\infty} \dfrac{\lfloor n}{n^n} z^n$ (ii) $\sum_{n=0}^{\infty} \dfrac{(\lfloor n)^2}{(2n)!} z^n$

9. (a) Prove that the series $\sum u_n(x) v_n(x)$ will be uniformly convergent on [a, b] if

   (i) $(v_n(x))$ is a positive monotonic decreasing sequence converging uniformly to zero for $a \le x \le b$.

   (ii) $|v_n(x)| = \left| \sum_{v=1}^{n} u_r(x) \right| < k$ for every value of x in [a, b] and for all integral value of n where k is a fixed number independent of x.

   (b) Show that the sequence (f_n) where $f_n(x) = \dfrac{x}{1 + nx^2}$ converges uniformly on R.

10. If $u_1, u_2, u_3, \ldots\ldots\ldots, u_n$ are functions of $y_1, y_2, \ldots\ldots., y_n$ and $y_1, y_2, y_3 \ldots\ldots., y_n$ are functions of $x_1, x_2, x_3 \ldots\ldots., x_n$ then show that

$$\frac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(x_1, x_2, \ldots.., x_n)} = \frac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(y_1, y_2, \ldots.., y_n)} \cdot \frac{\partial(y_1, y_2, \ldots.., y_n)}{\partial(x_1, x_2, \ldots.., x_n)}.$$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that a necessary and sufficient condition for a Set $S \subseteq R^k$ to be L-measurable is that for every Set $W \subseteq R^k$

$$|W| = |W \cap S| + |W \cap S^c|$$

i.e. $$m(W) = m(W \cap S) + m(W \cap S^c)$$

(b) If $A_1, A_2, A_3, \ldots\ldots\ldots A_n$ are L-measurable mutually disjoint sets in R^k then show that

(i) $$W \subseteq R^k \Rightarrow \left| W \cap \sum_{r=1}^{n} A_r \right| = \sum_{r=1}^{n} |W \cap A_r|$$

(ii) $$m\left(\sum_{r=1}^{n} A_r\right) = \sum_{r=1}^{n} m(A_r)$$

2. (a) Let (A_r) be a sequence of L-measurable sets such that

(i) $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots\ldots\ldots$ and $A = \bigcap_{r=1}^{\infty} A_r$

(ii) $m(A_1) < \infty$

then $$m(A) = \underset{r \to \infty}{lt}\ m(A_r)$$

3. If A, B, C are L-measurable sets. Then show that
$$m(A) + m(B) + m(C) = m(A \cup B \cup C) + m(B \cap C) + m(C \cap A) + m(A \cap B) + m(A \cap B \cap C)$$

4. (a) Prove that the class of L-measurable functions is closed with respect to all arithmetical operations.

(b) Prove that a necessary and sufficient condition for a function f to be measurable is that it is the limit of a convergent sequence of simple functions.

5. (a) If E and F are measurable sets and f is a integrable function on E + F then show that
$$\int_{E+F} f d\mu = \int_E f d\mu + \int_F f d\mu$$

(b) If f is L-integrable over X then prove that $\left| \int_X f d\mu \right| \le \int_X |f| d\mu$.

6. State and prove bounded convergence theorem.

7. Let (f_n) be a non-decreasing sequence of integrable functions defined over a measurable Set E and let $\lim_{n \to \infty} f_n$ be integrable over E then $\lim_{n \to \infty} \int_E f_n d\mu = \int_E \lim_{n \to \infty} f_n d\mu$.

8. State Lebesgue's dominated convergence theorem and use it to evaluate the following integral. $\lim_{n \to \infty} \int_0^1 f_n(x) dx$ where $f_n(x) = \dfrac{n^{3/2} x}{1 + n^2 x^2}$.

9. Verify bounded convergence theorem for $f_n(x) = \dfrac{nx}{1 + n^2 x^2}$.

10. (a) Prove that every absolutely continuous function is an indefinite integral of its own derivative.

(b) Let x be a Lebesgue point of a function f(t) then show that the indefinite integral

$$F(x) = f(a) + \int_a^x f(t) dt.$$

is differentiable at each point x and $F'(x) = f(x)$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Define a Topological space, Indiscrete Topology, Discrete topology, co-finite topology and co-countable topology.
   (b) Let $\{T_i : i \in I\}$ where I is an arbitrary set, be a collection of topologies for X. Then show that the intersection $\cap\{T_i : i \in I\}$ is also a topology for X.
2. Define a metrizable space and Equivalent metrics giving one suitable example for each.
3. (a) What do you mean by T_2-space or a Hausdorff space. Prove that every discrete space is a Hausdorff space.
   (b) Define the closure of a Set $A \subseteq X$ where (X, T) is a topological space. If (X, T) is a topological space and A and B are any two subsets of X then prove that,
   (i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ (iii) $\overline{\overline{A}} = \overline{A}$
4. (a) Define a subspace of a topological space (X, T). Let (X, T) be a topological space and $Y \subseteq X$. Then show that the collection $T_Y = \{G \cap Y : G \in T\}$ is a topology on Y.
   (b) What do you mean by Hereditary property of a topological space (X, T). Let (Y, T_Y) be a sub space of (X, T) and let B be a base for T. Then show that $B_Y = \{B \cap Y : B \in \mathrm{B}\}$ is a base for T_Y.
5. (a) Let (X, T_1), (Y, T_2) be two topological spaces. Then prove that a mapping $f : X \to Y$ is $T_1 - T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.
   (b) Let (X, T_1), (Y, T_2) be two topological spaces. Then prove that a mapping $f : X \to Y$ is closed if and only if $f(\overline{A}) \supset \overline{f(A)}\ \forall\ A \subseteq X$.
6. (a) How do you define a homeomorphism between two topological spaces. If (X, T_1), (Y, T_2) are two topological spaces and let $f : X \to Y$ which is one-one, onto then prove that the following statements are equivalent.
   (i) f is a homeomorphism.
   (ii) f is continuous and open.
   (iii) f is continuous and closed.
   (b) If (X, T_1), (Y, T_2) are two topological spaces and let $f : X \to Y$ be one-one, onto. Then prove that f is homeomorphism iff $f(\overline{A}) = \overline{f(A)}\ \forall\ A \subseteq X$.
7. Prove that the union of any family of connected sets having a non-empty intersection is connected.
8. (a) Prove that every closed subset of a compact space is compact.
   (b) Prove that every compact sub space of a Hausdorff space is closed.
9. (a) Prove that every second countable space is separable.
   (b) Prove that a topological space (X, T) is a T_1-space iff every singleton subset {x} of X is a T-closed set.
10. (a) Prove that every convergent sequence in a Hausdorff space has unique limit.
    (b) Prove that every closed-subspace of a normal space is normal.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If W_1 and W_2 are finite dimensional sub spaces of a vector space V, then prove that $W_1 + W_2$ is also finite dimensional and
$\dim \cdot W_1 + \dim \cdot W_2 = \dim \cdot (W_1 \cap W_2) + \dim \cdot (W_1 + W_2)$.

   (b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3 and express each of the standard basis vectors as linear combinations of α_1, α_2 *and* α_3.

2. (a) Show that the set $\{x^2 + 1, 3x - 1, -4x + 1\}$ is linearly independent and the set $\{x + 1, x - 1, -x + 5\}$ is linearly dependent.

   (b) Prove that all bases for a vector space V have the same number of vectors.

3. (a) Prove that the row space and the column space of a matrix A have the same dimension.

   (b) Find the basis for the row space of the following matrix A and determine its rank $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 1 & 5 \end{bmatrix}$.

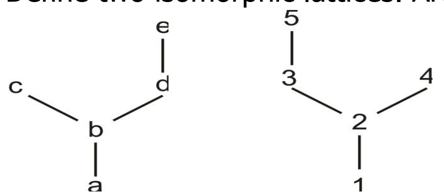
4. Let $T : U \to V$ be a linear transformation. Then prove that
$\dim \cdot \ker(T) + \dim \cdot \mathit{range}(T) = \dim \cdot \mathit{domain}(T)$

5. Prove that a linear transformation E on a linear subspace L is a projection on some subspace if and only if it is idempotent i.e. $E^2 = E$.

6. (a) Define a Lattice and the dual of a statement in a Lattice. Give two examples to make it clear.

   (b) Define a sub lattices with two examples.

7. (a) Define two isomorphic lattices. Are the two lattices shown in the figure are isomorphic ?



   (b) Define a bounded lattice and show that every finite lattice is bounded.

8. (a) What do you mean by a complemented lattice. Prove that if L be a bounded distributive lattice then complements are unique if they exist.

   (b) If L is a complemented lattice with unique complements then the join irreducible elements of L other than O are its atom. Prove it.

9. (a) Consider the Boolean algebra D_{210} the divisors of 210. Find the number of sub algebras of D_{210}.

   (b) Find the maxiterms of the Boolean algebra $P(A)$ consisting of the subsets of $A = \{a, b, c\}$.

10. (a) Express $E = Z(x' + y) + y'$ in complete sum of products form.

    (b) Write the Boolean expression $E(x, y, z)$ first as a sum of products and then in complete sum of products form.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Derive necessary and sufficient condition for $f(z)$ to be analytic in polar co-ordinates.

   (b) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|R f(z)|^2 = 2|f'(z)|^2$.

2. (a) Examine the behaviour of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^2}$ on the circle of convergence.

   (b) Find the domain of convergence of the series $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5 ....(2n-1)}{n!}\left(\frac{1-z}{2}\right)^n$.

3. (a) Find the bilinear transformation which maps the points $z = -2, 0, 2$ into points $w = 0, i, -i$ respectively.

   (b) Show that the transformation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle.

4. State and prove Cauchy integral formula i.e. If $f(z)$ is analytic within and on a closed contour C and if a is any point within C then $f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)}{z-a}\cdot dz$.

5. State and prove poison's integral formula.

6. State and prove Laurent' theorem.

7. Find the Laurent's series of the function $f(z) = \frac{1}{z^2(1-z)}$ about $z = 0$ and expand $\frac{1}{z^2 - 3z + 2}$ for (i) $0 < |z| < 1$ (ii) $1 < |z| < 2$ (iii) $(z) > 2$.

8. State and prove maximum modulus principle.

9. (a) Discuss the nature and singularities of the following functions

   (i) $\frac{1}{z(z-1)^2}$ (ii) $\frac{z}{1+z^4}$ (iii) $\frac{1}{z(e^z - 1)}$

   (b) Evaluate the residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

10. Evaluate any two of the following integrals :–

    (a) $\int_0^{\pi} \frac{a\, d\theta}{a^2 + Sin^2\theta}, a > 0$ (b) $\int_0^{2\pi} \frac{Cos\, 2\theta}{5 + 4 Cos\theta} do$

    (c) $\int_0^{\infty} \frac{Cos\, mx}{x^2 + a^2} dx$ (d) $\int_0^{\infty} \frac{\log(1 + x^2)}{1 + x^2} dx$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Define a linear system and show that it satisfies Lipschitz condition and the set of solutions form a vector space.

2. Show that the function given below satisfy Lipschitz condition in the rectangle indicated and hence find Lipschitz constant $f(x, y) = (y + y^2)\frac{Cos\ x}{2}$, $|y| \le 1$, $|x - 1| \le ½$.

3. Compute the first three successive approximations for the solution of the Initial Value problem $y^1 = y^2$, $y(0) = 1$.

4. State and prove Ascolli's Lemma.

5. State and prove Cauchy-Peano existence theorem.

6. Find the nature of the critical point (0, 0) of the system of equations $\frac{dx}{dt} = 3x + 4y$ and $\frac{dy}{dt} = 3x + 2y$.

7. Test the stability of the non-linear system $\frac{dx}{dt} = x + 4y - x^2$, $\frac{dy}{dt} = 6x - y + 2xy$. Further make a comment on stability.

8. (a) What is the meaning of generating function for Legendre polynomial ? Hence find it.

   (b) Describe orthogonal property of Laguerre polynomial.

9. Find the series solution of Bessel's differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$.

10. (a) Solve the differential equations by matrix method

    $$\frac{dx_1}{dt} = 9x_1 - 8x_2$$

    $$\frac{dx_2}{dt} = 24x_1 - 19x_2$$

    The initial conditions for which are $x_1(0) = 1$, $x_2(0) = 0$.

    (b) Define fundamental matrix and show that a necessary and sufficient condition that a solution matrix G to be fundamental matrix is $G(x) \neq 0$ for $x \in I$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–VIII

(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If α, β, λ are any three Cardinal numbers then prove that
   (i) $\alpha + (\beta + \lambda) = (\alpha + \beta) + \lambda$ (ii) $\alpha(\beta + \lambda) = \alpha\beta + \alpha\lambda$
   (b) Prove that every set can be well ordered.
2. (a) Prove that the set of all real numbers R is uncountable.
   (b) If A and B are countable sets then show that A × B is also countable.
3. (a) Prove that Zorn's lemma implies well ordering theorem.
   (b) What is Axiom of choice ? Show that the Axiom of choice is equivalent to Zermelo's postulates.
4. (a) State and prove division algorithm in theory of numbers.
   (b) Find g.c.d. of 28 and 49 and express it as a linear combination of 28 and 49.
5. (a) If a_n is the nth term of the Fibonaced sequence and $\alpha = \frac{1 + \sqrt{5}}{2}$ then $a_n > \alpha^{n-1} \; \forall \; n > 1$.
   (b) Factorize 493 by Euler's Factorization method.
6. (a) Solve the congruence $x^3 \equiv 5 \pmod{13}$.
   (b) Show that $T(n) = \left[\frac{n}{1}\right] + \left[\frac{n}{2}\right] + \left[\frac{n}{3}\right] + \dots\dots + \left[\frac{n}{n}\right]$.
7. (a) Prove that an undirected graph is a tree iff there is a unique path between any two vertices.
   (b) If a tree has n vertices of degree 4 then find the value of n.
8. (a) Show that a complete graph of n vertices is a planner if $n \le 6$.
   (b) If a tree has n vertices of degree 1, two vertices of degree 2, four vertices of degree 4 then find the value of n.
9. Define an Umbilic. Prove that in general three lines of curvature pass through an umbilic.
10. Show that for a geodesic

    $T^2 = (K - K_1)(K - K_2)$ where

    K is curvature and T is torsion.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Show that $\Delta^n x^{(n)} = \underline{|n}\, h^n$ and $\Delta^{n+1} x^{(n)} = 0$.

   (b) Express $2x^3 - 3x^2 + 3x - 10$ and its difference in factorial notation, the interval of differencing being unity.

2. (a) Use the method of separation of symbols to prove that
   $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \ldots\ldots\ldots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n-1}$.

   (b) Obtain the estimate of the missing numbers in the following table.

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	?	64	?	216	343	512

3. Prove that the divided differences can be expressed as the product of multiple integrals.

4. Find the polynomial of fifth degree from the following data
   $u_0 = -18,\ u_1 = 0,\ u_3 = 0,\ u_5 = -248,\ u_6 = 0,\ u_9 = 13140$.

5. Find $f'(7.50)$ from the following table.

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
$y = f(x)$	.193	.195	.198	.201	.203	.206	.208

6. Find the value of $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rd and $\frac{3}{8}$ th rule, hence obtain the approximate value of π in each case.

7. (a) By using the method of iteration find a real root of $2x - \log_{10}^{x} = 7$.

   (b) Find the root of the equation $x^3 - 6x - 11 = 0$ which lies between 3 and 4.

8. (a) Find the sum to n terms of the series whose x^{th} term is $2^x\left(x^3 + x\right)$.

   (b) Solve the equation $y_{h+2} - 4y_{h+1} + 4yh = 0$.

9. (a) Use synthetic division to solve $f(x) = x^2 - 1.0001x + 0\cdot 9999 = 0$ in the neighbourhood of x = 1.

   (b) Solve the equation $3x - Cosx - 1 = 0$ by false position method and Newton Raphson method.

10. (a) Obtain an approximation in the sense of the principle of least squares in the form of polynomial of second degree to the function $f(x) = \frac{1}{1+x^2}$ in the range $-1 \le x \le 1$.

    (b) Fit a second degree parabola to the following :–

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

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Revised Examination Programme, 2018

M.Sc. Mathematics, Part-II

Date	***Paper***	***Time***	***Examination Centre***
14.06.2018	Paper–IX	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
18.06.2018	Paper–X	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
20.06.2018	Paper–XI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
22.06.2018	Paper–XII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
26.06.2018	Paper–XIII	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
28.06.2018	Paper–XIV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
30.06.2018	Paper–XV	12.00 Noon to 3.00 PM	Nalanda Open University, Patna
02.07.2018	Paper–XVI	12.00 Noon to 3.00 PM	Nalanda Open University, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If x and y are any two vectors in an inner product space then prove that $|(x,\ y)| \le \| x \| \cdot \| y \|$.

   (b) Show that the inner product space is jointly continuous.

2. State and prove Riesz Lemma.

3. State and prove closed graph theorem.

4. (a) Show that a normal linear space is a metric space under the property $\big|\| x \| - \| y \|\big| \le \| x \| - \| y \|$.

   (b) If x and y are any two vectors on a Hilbert space H then show that $\| x - y \|^2 + \| x + y \|^2 = 2\left(\| x \|^2 + \| y \|^2\right)$.

5. Consider a real number p such that $1 \le p < \infty$. Denote l_p the space of all sequences $x = (x_1,\ x_2,\ x_3,\ \ldots\ldots\ldots\ldots)$ of scalars such that $\sum_{n=1}^{\infty} | x_n | p < \infty$ with norm defined by $\| x \|_p = \left(\sum_{n=1}^{\infty} | x_n | p\right)^{1/p}$. Show that l_p is a Banach space.

6. If $f,\ g \in L^p(a,\ b)$ where p > 1 then prove that $\| f + g \|_p \le \| f \|_p + \| g \|_p$.

7. If N and N' be normal linear spaces and T be a linear transformation of N into N'. Then show that the following conditions are equivalent to one another.
   (i) T is continuous.
   (ii) T is continuous at the origin (i.e. $x_n \to 0 \Rightarrow T(x_n) \to 0$)
   (iii) T is bounded (i.e. $\exists$ a real number $K \ge 0\ \ s \cdot t\ \ \|T(x)\| \le K \| x \|\ \forall\ x \in N$).
   (iv) If $S = \{x : \| x \| = 1\}$ is closed sphere in N then the image T(S) is a bounded set in N'.

8. If N and N' be normal linear spaces and let T : N $\to$ N' be any linear map. If N is finite dimensional then prove that T is continuous or bounded.

9. State and prove open mapping theorem.

10. (a) Prove that if M and N are closed linear sub spaces of a Hilbert space H $s \cdot t\ \ M \perp N$, then M + N is also closed.

    (b) If T is a normal operator on a Hilbert space H then prove that $\| T^2 \| = \| T \|^2$.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XI

(Partial Differential Equations)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Find the complete integral of $\left(p^2 + q^2\right)x = pz$. (use Charpit's method)
   (b) Solve $p^2 x + q^2 y = z$ by Jaicobi's method.

2. Solve
   (a) $\left(D^2 + 2DD' + D'^2\right)z = e^{3x + 2y}$
   (b) $\left(D^2 + 3DD' + 2D'^2\right)z = x + y$

3. Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution.

4. (a) Solve $pt - qs = q^3$, by Monge's method.
   (b) Solve $(q + 1)s = (p + 1)t$ by Monge's method.

5. (a) Find the integral surface of the equation $(2xy - 1)p + \left(z - 2x^2\right)q = 2(x - yz)$ which passes through the line $x = 1,\ y = 0,\ z = 1$.
   (b) Find the characteristics curve of $2yu_x + \left(2x + y^2\right)u_y = 0$ passing through (0, 0).

6. Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

7. Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\left(\frac{\partial u}{\partial t}\right)$ satisfying the conditions
   $0 = u(0, t) = u(l, t),\ u(x, 0) = \left(x - x^2\right)$.

8. Show that the general solution of the wave equation $c^2\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial^2 u}{\partial t^2}$ is $u(x, t) = \phi(x + ct) + \psi(x - ct)$ where ϕ and ψ are arbitrary functions.

9. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(x, 0) = u(x, b) = 0$ for $0 \le x \le a$, and $u(a, y) = f(y)$ for $0 \le y \le 6$.

10. Transform the Laplaces equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in the cylindrical co-ordinates.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XII
(Analytical Dynamics)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Explain the principle of least action and hence establish it in terms of arc length of a particle path.
   (b) A particle moves in a plane under a central force depending on its distance from the origin. Then construct the Hamiltonian of the system and derive Hamilton's equation of motion.

2. State and prove Jaicobi-Poisson theorem.

3. Determine the kinetic energy and the moment of momentum of a rigid body rotating about a fixed axis.

4. (a) Prove that in a simple dynamic system $T + V =$ constant where T and V have their usual meaning.
   (b) Derive the formula for the Kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of Kinetic energy.

5. Derive Euler's equation of motion for the motion of rigid body about a fixed point.

6. (a) State and prove Jaicobi-theorem.
   (b) A particle of mass m moves in a force field whose potential in spherical co-ordinates is given by $V = \frac{\lambda \, Cos\, \theta}{r^2}$, then write down the Hamilton Jaicobi equation and derive the complete solution.

7. (a) Show that the transformation $Q = q \tan p$, $P = \log(Sin\ p)$ is canonical.
   (b) Define the generating function of a transformation and give an example of a generating function of transformation.

8. (a) Derive Lagrange's equation of impulsive motion in a Holonomic dynamical system.
   (b) A bead is sliding on a uniformly rotating wire in a force free space. Derive its equation of motion.

9. (a) Construct Routhian function and Routh's equation for the solution of a problem involving cyclic and non-cyclic co-ordinates.
   (b) Use Routhian equation of motion to determine the motion of a uniform heavy rod turning about one end which is fixed.

10. (a) Explain the terms (i) degree of freedom, (ii) Constraints, (iii) generalized co-ordinates and classify the dynamical systems based on different types of constraints.
    (b) Explain the difference between possible displacement and virtual displacement. Give one example of each.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIII
(Fluid Mechanics)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Derive Euler's equation of Fluid motion.
2. State and prove Kelvin's circulation theorem.
3. Obtain the equation of continuity in spherical polar co-ordinates.
4. Derive Cauchy Riemann differential equation in polar form.
5. (a) What do you mean by source and sink ? Find the complex potential due to a source of strength m placed at the origin.
   (b) Show that $u = Axy$, $v = A\,(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function of fluid motion.
6. (a) A two dimensional flow field in given by $\psi = xy$, then (i) show that the flow is irrotational (ii) Find the velocity potential (iii) find the stream lines and potential lines.
   (b) Show that the two dimensional irrotational motion, stream function satisfies Laplace's equation.
7. Obtain the boundary layer equations in two dimensional flow.
8. Derive the equation of energy for an incompressible fluid motion with constant fluid properties.
9. Show that the vorticity vector Ω of an incompressible viscous fluid moving under no external forces satisfies the differential equation.

   $\frac{D\Omega}{Dt} = (\Omega \bullet \nabla)\,q + v\ \nabla^2\ \Omega$ Where v is the kinematic of viscosity.
10. (a) Find the principal stresses and principal directions of stress at a point (1, 1, 1) if the components of the stess tensor are given by

$$\sigma\, y = \begin{pmatrix} 0 & 2y_3 & 0 \\ 2y_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

   (b) What type of motion do the following velocity components constitute ?
   $u = a + by - cz$, $\vartheta = d - bx + ez$, $w = f + cx - ey$ where a, b, c, d, e, f are arbitrary constants.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIV
(Operation Research)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Using simplex method solve L.P.P.
   Max $z = 4x_1 + 10x_2$
   Subject to the condition $2x_1 + x_2 \le 50,\ 2x_1 + 5x_2 \le 10,\ 2x_1 + 3x_2 \le 90;\ x_1 \ge 0,\ x_2 \ge 0$.
2. (a) Find basic feasible solution of the system $2x_1 + x_2 + 4x_3 = 11,\ \ 3x_1 + x_2 + 5x_3 = 14$.
   (b) Reduce feasible solution $x_1 = 2,\ x_2 = 4,\ x_3 = 1$ of the system $2x_1 - x_2 + 2x_3 = 2$ and $x_1 + 4x_2 = 18$ to a basic feasible solution and mention its kind (degenerate or non-degenerate).
3. Explain in details the dual simplex method elaborating each step.
4. (a) Obtain the feasible solution of non-linear programming
   Min $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
   Subject to $x_2 \le 8,\ \ x_1 + x_2 \le 10,\ \ x_1 \ge 0,\ x_2 \ge 0$.
   (b) By using Lagrange's multiplier method solve the NLPP
   $z = a\,x_1^2 + b\,x_2^2 + c\,x_3^2$ where $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$.
5. (a) Describe the method of constructing the solution of 'Game Problem' where the game is without saddle point.
   (b) Solve the game problem whose pay off matrix is $\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$.
6. Apply two phase method to compute the solution of
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \ge 4,\ \ x_1 + 7x_2 \ge 7;\ \ \ x_1 \ge 0,\ \ x_2 \ge 0$.
7. (a) Define hyper plane and hyper sphere. Prove that every hyper plane in R^n is a convex set.
   (b) Prove that the intersection of any finite number of convex sets is a convex set.
8. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   S.t. $x_1 - 3x_2 + 4x_3 = 5,\ \ x_1 - 2x_2 \le 3,\ \ 2x_2 - x_3 \ge 4;\ \ x_1, x_2 \ge 0;\ x_3$ is unrestricted in sign.
9. Solve the following assignment problem.

		Man I	II	III	IV	V
Task	A	1	3	2	3	6
	B	2	4	3	1	5
	C	5	6	3	4	6
	D	3	1	4	2	2
	E	1	5	6	5	4

10. Solve the following transportation problem.

	To 1	2	3	Supply
From 1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV
(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that the inner product of tensors A_q^p and B_i^{ij} is a tensor of rank three.
   (b) Define the inner and outer product of two tensors and prove that the outer product of two tensors is a tensor of rank equal to the sum of ranks of the two tensors.
2. (a) Show that any linear combination of tensors of type (r, s) is a tensor of type (r, s).
   (b) Prove that a skew symmetric tensor of rank two has $\frac{N}{2}(N-1)$ independent components.
3. (a) What do you mean by Christoffel's symbols and prove that
   $[ij, k] + [jk, i] = \frac{\partial g_{ik}}{\partial x^j} = \partial_j g_{ik}$.
   (b) Show that $\left\{ \begin{matrix} i \\ i\ j \end{matrix} \right\} = \frac{\partial \log(\sqrt{g})}{\partial x^j} = \frac{\partial \left(\log\sqrt{-g}\right)}{\partial x^j}$.
4. Find the Laplace transform of
   (a) $L\left\{Sin\sqrt{t}\right\}$ (b) $L\left\{Sin^2 at\right\}$ (c) $L\left\{e^{at}\ Cos\ bt\right\}$ (d) $L\left\{e^{-t}\ Cos^2\ t\right\}$
5. (a) Find (a) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ (b) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$
6. Using Laplace transform solve the following differential equations
   (a) $(D+1)^2 y = t$, given that $y=-3$, when t = 0 and y = -1 when t = 1.
   (b) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 2\,Sin\,t$, given that $y = \frac{dy}{dt} = 0$ when t = 0.
7. (a) Find the Fourier cosine transform of e^{-x^2}.
   (b) Find the Fourier sine transform of $x\,e^{-\frac{x^2}{2}}$.
8. (a) Find Fourier transform of $F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence prove that $\int_0^\infty \frac{Sin^2\,ax}{x^2}dx = \frac{\pi a}{2}$.
   (b) State and prove convolution theorem of Fourier transforms.
9. (a) Describe Firedholm integral equation and voltera integral equation.
   (b) Explain about the Fredholm integral equations of three kinds.
10. The maintenance and re-sale value per year of a machine whose purchase prices is Rs. 7000/- is given below :—

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Re-sale Value in Rs.	4000	2000	1200	600	50	400	400	400

When should the Machine be replaced.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2018

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What are the data types in C language ? Distinguish between constant and variable.
2. What is an Operator ? Describe different types of operators in C with examples.
3. What is recursion ? Write a program to find the roots of a quadratic equation.
4. What are reserved words ? What is the difference between the expression "++a" and "a++" ? Explain with examples.
5. What is an Array ? How does an Array differ from an ordinary variable ?
6. What are header files and what are its uses in C programming ? What is the difference between the = symbol and == symbol ?
7. What are logical errors and how does it differ from syntax errors ? Write a program in C to swap the value of two variables.
8. What is debugging ? When is the "void" keyword used in a function ?
9. Write short notes on the following :—
   (i) Micro Substitution
   (ii) Conditional operator
   (iii) Branching statements
   (iv) Pointer to function.
10. Write a program in C to check whether a given number is a Prime Number.

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2018

Practical Counselling Programme

Enrollment No.	***Date***	***Time***	***Venue***
160290001 to 160290590	03.07.2018 to 09.07.2018	9.00 AM to 1.00 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
160290591 to 160290922 & All Old Students		1.00 PM to 5.00 PM	

Practical Examination Programme

Enrollment No.	***Date***	***Time***	***Venue***
160290001 to 160290240	10.07.2018	11.00 AM to 2.00 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
160290241 to 160290590	10.07.2018	2.30 PM to 5.30 PM	
160290591 to 160290922	11.07.2018	11.00 AM to 2.00 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
All Old Students	11.07.2018	2.30 PM to 5.30 PM	