Nalanda Open University

B.Sc Part-II

Course: Physics

Paper- III

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Topic – Zone Plate and Comparision of zone plate with convex lens.

Zone Plate : The working of the zone plate provides an experimental verification of half period zone construction given by Fresnel. The half period zones, which are drawn on the wave front, are out of phase successively at a point of observation.

Construction:- In order to prepare the zone plate, concentric circle whose radii are proportional to the square root of natural numbers are drawn on a white sheet of paper. The alternate zones formed in this way are painted blank. A negative of a highly reduced photograph of this drawing is called a zone plate.

Types of zone plates:-

(i) **Negative zone plate**- For preparing negative zone plate second, fourth, sixth. i.e. even zones are painted black in the drawing, the reduced negative photograph of this drawing will provide negative zone plate.

(ii) **Positive zone plate**- For preparing positive zone plate, first, third, fifth, ... i.e. odd zones are painted black in the drawing, the reduced negative photograph of this drawing will provide positive zone plate.

Zone Plate has multiple focii:-

Theory:-

Let YY′ be a transparent screen on a normal to it is situated at point O emitting waves of wave length ƛ. We have to find the intensity at the point P, laying on the same normal to screen but on the other side of the screen. Now divide the screen into half period zones with Q as pole. These zones intersect the screen at M1, M2, M3, M4....etc.

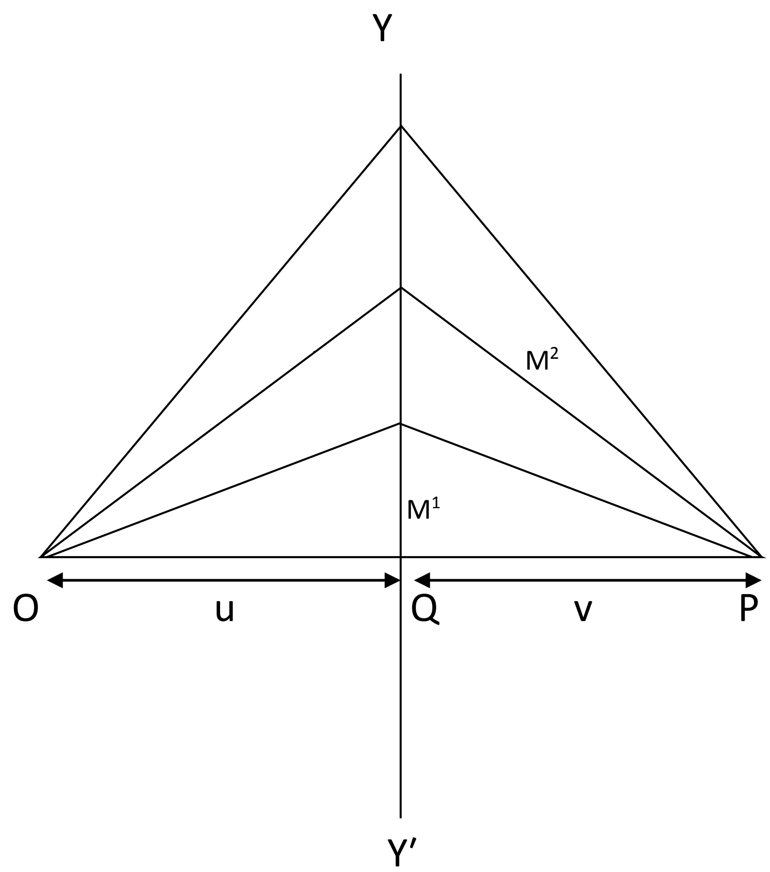

Figure: 1

Obviously $OM_nP – OQ.P = n\lambda/2$

Let the radius of the n^{th} circle around Q be given by r_n.

From the $\Delta\ OM_nQ$

$$OM_n = (OQ^2 + QM_n^2)^{1/2}$$

$$OM_n = (u^2 + r_n^2)^{1/2} \qquad \text{(Since } OQ = u \text{ and } QM_n = r_n)$$

$$OM_n = u(1+r_n^2/u^2)^{1/2}$$

Using Bionomial theorem

$$OM_n = u(1+r_n^2/u^2)^{1/2} = u+r_n^2/2u$$

Similarly, from the $\Delta\ PM_nQ$

$$PM_n^2 = PQ^2+QM_n^2$$

$$PM_n = (PQ^2+QM_n^2)^{1/2}$$

$$= (v^2+r^2_n)^{1/2} \qquad (PQ=V \text{ and } QM_n=r_n)$$

$$PM_n = v\ (1+r_n^2/v^2)^{1/2} = v\ (1+1/2\ r_n^2/v^2) \quad \text{using Binomial theorem}$$

$$= v+1/2\ r_n^2/v$$

$$OM_nP = OM_n + M_nP$$

$$= u + r_n^2/2u + v + r_n^2/2v = u + v + ½\ (r_n^2/u + r_n^2/v)$$

Also $OQP = OQ + QP = u + v$

Hence, $OM_nP – OQP = u + v + ½\ (r_n^2/u + v_n^2/v) – u – v$
$= ½\ (r_n^2/u + r_n^2/v)$

But, $OMnP – OQP = n\lambda/2$

Hence, $r_n^2/2\ (1/u + 1/v) = n\lambda/2$

$$r_n^2 = n\lambda / (1/u + 1/v) = uvn\lambda / u+v$$

Therefore , $r_n = k(n)^{½}$ where $\frac{(uv\lambda)}{u+v} = K$

or , $r_n\ \alpha\ (n)^{1/2}$

i.e., the radii of the circles are proportional to the square root of the natural numbers.

Hence, if the alternate zones of the screen $yy^{/}$ be made opaque, it will become a zone plate. Since the path difference between the waves from alternate zones is λ the waves from all the transparent zones reaching at P will be in phase. If the second, fourth, sixth ..... i.e. even zones are opaque the amplitude reaching at P from the illuminate zone plate.

= R1+R3+R5+R7+R9..............

If first, third, fifth ....i.e. the odd zones are opaque, the amplitude reaching P, from the illuminated zone plate

= - R2 – R4 – R6 – R8 – R 10 ....

= - (R2 + R4 + R6 + R8 + R 10 ....)

Thus, the point P will be sufficiently bright, hence, it is called the image of O. Obviously, the points O and P act as the conjugate points which are related by the equation

$$1/v+1/u = n\lambda/r_n^2 = 1/\ r_n^2/\ n\lambda$$

Obviously, the focal length of the zone plate is given by

$$f = r_n^2/\ n\lambda$$

Comparison of Zone Plate with Convex lens:**-** The Zone plate and Convex lens are similar in the following respects.

1) Both of them form real images on the side other than that of the object.
2) The formula connecting the conjugate distance(i.e. u,v) are of similar type.
3) The focal length of both varies with the wave length (λ) hence both form coloured image with the help of white light.
4) Magnification is possible with both device.

. **The zone plate and Convex lens are dissimilar in the following respects**.

1) The focal length of the lens varies directly as the wave length. As the wave length of the red colour is greater than that of the violet hence the focal length of the red is greater than that of the violet i.e. $Fr > Fv$

But in case of the zone plate, the focal length varies inversely as the wave length , hence , $Fr < Fv$

Hence the chromatic aberration produced by zone plate and convex lens are of opposite sign with respect to each other.

2) The waves from different zones of a zone plate travel different optical paths as they reach the image point. While in the Convex lens all the waves travel equal optical path. The successive waves coming at P after transversing the zone plate differ in path by λ or in phase by T (time period), whereas the path difference between the waves reaching at P1 through lens in zero.
3) The intensity of the image formed by the zone plate is less than that formed by the lens.
4) Zone plate forms the image of the point by virtue of diffraction whereas, lens forms the image by virtue of refraction.
5) In case of the lens we have only one focus while for a zone plate, we have a number of foci, whose intensity decreases successively.

When the distance of the point P from the zone plate (i.e. v) is such that each of our constructed zone on our plate coincides with the theoretical Fresnel's zone, formed on the incident wave front, the image in intense.

Now if the distance v is increased, a position will be reached when each of our constructed zone, contains three theoretical h.p. zones, hence the resultant amplitude at the point of observation.

$= R_1 - R_2 + R_3 + R_7 - R_8 + R_9 + R_{13} - R_{14} + R_{15}$

$= \frac{R_1}{2} + \frac{R_3}{2} + \frac{R_7}{2} + \frac{R_9}{2} + \frac{R_{13}}{2} + \frac{R_{15}}{2}$

$= \frac{1}{2} (R_1 + R_3 + R_7 + R_9 + R_{13} + R_{15})$

= A maxima.

Now if v is decreased so that each of our zone drawn on plate contains 5 half period zones of the wave front, then the resultant amplitude reaching the point of observation.

$= R_1 - R_2 + R_3 + R_4 - R_5 + R_{11} - R_{12} + R_{13} - R_{14} + R_{15} + \ldots\ldots\ldots\ldots$

$= \frac{1}{2} (R_1 + R_5 + R_{11} + R_{15} + \ldots\ldots\ldots\ldots)$

= A maxima .

In this way we shall get a number of foci of decreasing intensity.