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Department of Economics

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Course – MA Economics (Part –I)
Paper – III (Quantitative Methods)
Topic – Differentiation

Differentiate the following

(i) $x\sqrt{y} + y\sqrt{x} = 1$

Soln –

$$x\sqrt{y} = 1 - y\sqrt{x}$$

Differentiating the term w. r. t. x we have,

$$d/dx [(y\sqrt{x})] = d/dx (1 - x\sqrt{y})$$

$$\sqrt{x} \frac{dy}{dx} + y (d\sqrt{x}/dx) = - [x (d\sqrt{y}/dx) + \sqrt{y} (dx/dx)]$$

$$\sqrt{x} \frac{dy}{dx} + \frac{1}{2} x^{-1/2} y = - x \frac{1}{2} y^{-1/2} \frac{dy}{dx} - \sqrt{y}$$

$$\sqrt{x} \frac{dy}{dx} + \frac{y}{2\sqrt{x}} = - \frac{x}{2\sqrt{y}} \frac{dy}{dx} - \sqrt{y}$$

$$[\sqrt{x} + \frac{x}{2\sqrt{y}}] \frac{dy}{dx} = - \frac{y}{2\sqrt{x}} - \sqrt{y}$$

$$\frac{x + 2\sqrt{xy}}{2\sqrt{y}} \frac{dy}{dx} = - [\frac{y + 2\sqrt{xy}}{2\sqrt{x}}]$$

$$\frac{dy}{dx} = - [\frac{y + 2\sqrt{xy}}{2\sqrt{x}}] [\frac{2\sqrt{y}}{x + 2\sqrt{xy}}]$$

$$\frac{dy}{dx} = - \left[\sqrt{\frac{y}{x}} \right] \left[\frac{y+2\sqrt{xy}}{x+2\sqrt{xy}} \right] \text{ Ans.}$$

(ii) $x = y \log(xy)$

Differentiating the term w. r. t. y we have,

$$\begin{aligned} \frac{dx}{dy} &= \log(xy) \frac{dy}{dy} + y \frac{d \log(xy)}{dy} \\ &= \log(xy) + y \left[\frac{1}{xy} \right] \left[\frac{d(xy)}{dy} \right] \\ &= \log(xy) + \left[\frac{1}{x} \right] \left[y \frac{dx}{dy} + x \frac{dy}{dy} \right] \\ &= \log(xy) + \frac{1}{x} \left[y \frac{dx}{dy} + x \right] \\ &= \log(xy) + 1 + \left[\frac{y}{x} \frac{dx}{dy} \right] \end{aligned}$$

$$\frac{dx}{dy} - \left[\frac{y}{x} \frac{dx}{dy} \right] = \log(xy) + 1$$

$$\left[1 - \frac{y}{x} \right] \frac{dx}{dy} = \log(xy) + 1$$

$$\left[\frac{x-y}{x} \right] \frac{dx}{dy} = \log(xy) + 1$$

$$\frac{dx}{dy} = \frac{x [1 + \log(xy)]}{x-y} \text{ or,}$$

$$\frac{dy}{dx} = \frac{x-y}{x [1 + \log(xy)]} \text{ Ans.}$$